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Editor-in-Chief

MARTIN SUMMERFIELD

Optimum Thrust Programming for a Sounding Rocket

By H. S. TSIEN[1] and ROBERT C. EVANS[2]

The problem of optimum thrust programming for a sounding rocket of minimum starting weight to reach specified height with given final weight and propellant characteristics is first formulated as a problem in variational calculus. The general solution for arbitrary drag function is given. The solution is then applied to two special cases, one with quadratic drag dependence on velocity and the other with linear drag dependence on velocity. Complete numerical data are given. The results are then compared with the results of constant thrust to show the advantages of thrust programming. Thrust programming is shown to be able to increase appreciably the pay load of a high altitude sounding rocket.

FOR a rocket in vertical flight, the aerodynamic drag of the rocket body and the gravitational pull are in the same direction and directly opposite to the direction of the thrust force. The performance computation is thus relatively simple. It was indeed carried out by F. J. Malina and A. M. O. Smith (1).[3] They have shown that if the aerodynamic drag is absent, then the best way of using the propellant is to use it in the shortest possible time. Theoretically, a rocket of given weight fraction of propellant will reach the highest altitude if the thrust is applied as a single impulse and thus reaches maximum velocity immediately. This result can be understood, perhaps, by considering the other extreme of having the thrust equal to the weight of the rocket at every time instant. Then the rocket, having zero acceleration at all times, will not be able to leave the ground. Long drawn-out operation of the rocket, therefore, is definitely unprofitable if aerodynamic drag can be neglected.

When there is aerodynamic drag, the high velocity reached at low altitudes by an impulsive start will give a very high drag which tends to reduce the maximum altitude of the rocket. In fact, the calculations made by Malina and Smith, and more recently by Ivey, Bowen, and Oborny (2), show that, assuming constant thrust, the optimum initial acceleration is one to three g, depending on the ratio of drag and weight of the rocket. The more general problem, however, is that of optimum thrust programming, i.e., to find the optimum time variation of the thrust for maximum altitude. This is the theoretical optimum design. Practical design is complicated by the added weight on the power plant to make the thrust variable and certainly cannot reach the theoretical optimum condition. The theoretical optimum gives a standard of comparison and shows how much could be expected by varying the thrust.

The problem of optimum thrust programming was studied quite early by G. Hamel (3). He made the simplifying assumptions that the density of air decreases exponentially with altitude, and that the effective exhaust velocity c of the rocket motor and the gravitational constant g do not vary with altitude. Using variational calculus he gave the solution of the problem of optimum thrust programming. However, his paper is very brief and is not easy to understand. It is the purpose of the present authors to give a complete discussion of the problem together with more extensive numerical data.

Formulation of the Problem

Let M be the mass of the rocket and s the height at time instant t. Following Hamel, the effective exhaust velocity c of the rocket motor will be taken as a constant. The velocity of the rocket is ds/dt and will be denoted as $\dot{s}$. For a specified rocket body, the aerodynamic drag D is a function of the altitude s, and the velocity $\dot{s}$. If g is the constant value of the gravitational force per unit mass, the equation of motion of the rocket during the powered flight is

$$\frac{dM}{dt} + \frac{M}{c}\left\{\frac{d\dot{s}}{dt} + g\right\} = -\frac{D(s, \dot{s})}{c} \qquad [1]$$

The altitude s is measured from the point of launching. Therefore, at the beginning of powered flight, $t = 0$, $s = 0$, $M = M^0$, and $\dot{s} = \dot{s}_0$ where M^0 is the mass and $\dot{s}_0$ is the velocity after boosting. At the end of the powered flight, $t = t_1$, $M = M_1$, $s = s_1$, $\dot{s} = \dot{s}_1$. M_1 is then the final mass. Equation [1] together with these boundary conditions gives the following expression for mass M^0,

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[3] Numbers in parentheses refer to bibliography on page 107.

$$M^0 = \exp\left(-\frac{\dot{s}_0}{c}\right) \times$$
$$\left\{\int_0^{t_1} \frac{D(s, \dot{s})}{c} \exp\left(\frac{\dot{s} + gt}{c}\right)dt + M_1 \exp\left(\frac{\dot{s}_1 + gt_1}{c}\right)\right\} \quad [2]$$

If M_0 is the initial mass of the rocket including the propellant for boosting by a single impulse to the velocity $\dot{s}_0$, then

$$M_0 = M^0 \exp\left(\frac{\dot{s}_0}{c}\right) \quad [3]$$

Therefore, by combining [2] and [3],

$$M_0 = \int_0^{t_1} \frac{D(s, \dot{s})}{c} \exp\left(\frac{\dot{s} + gt}{c}\right) dt + M_1 \exp\left(\frac{\dot{s}_1 + gt_1}{c}\right) \quad [4]$$

Equation [4] does not explicitly contain the velocity after boosting, or initial velocity $\dot{s}_0$. If the initial velocity is zero, the acceleration of the rocket is smooth. If the initial velocity is not zero, the rocket starts with an impulse and then gradually accelerates. In any case, however, Equation [4] gives the total initial mass including the boosting charge.

Now let the problem of optimum thrust programming be formulated as follows: Given M_1, c, g, and the drag function $D(s, \dot{s})$, what should be the function $s(t)$, such that M_0 is a minimum? The auxiliary conditions are $s(0) = 0$ and that s_1 and $\dot{s}_1$ must be such as to reach the specified summit altitude S. To reach the given summit altitude with the specified M_1 and $D(s, \dot{s})$, s_1 and $\dot{s}_1$ are related, say

$$\dot{s}_1 = \phi(s_1) \quad [5]$$

where ϕ is a given function. For instance, at very high altitudes where the aerodynamic drag is negligible due to the low air density,

$$\dot{s}_1 \cong \sqrt{2g(S - s_1)} \quad [6]$$

To find the conditions for the solution of this variational problem, let the required s function be

$$s = s(t) \quad [7]$$

with $s(0) = 0$.

Now let there be an arbitrary function $\eta(t)$ such that

$$\eta(0) = 0 \quad [8]$$

but otherwise completely unspecified. Then the "neighboring" functions to $s(t)$ can be constructed as

$$\bar{s}(t) = s(t) + k(\epsilon)\eta(t) \quad [9]$$

where k is a parameter but not a function of time. Because of Equation [8], $\bar{s}$ satisfies the initial condition $\bar{s}(0) = 0$. The duration of powered flight, or the burning time, for the optimum solution is t_1. For the neighboring solution, the burning time is $t_1 + \epsilon$. Thus k is a function of ϵ. For the optimum solution, k and ϵ both vanish. Therefore,

$$\left.\begin{array}{l} k(0) = 0 \\ k(\epsilon) \cong \epsilon\, k'(0) \end{array}\right\} \quad [10]$$

By considering only terms up to first order in ϵ,

$$\bar{s}_1 = \bar{s}(t_1 + \epsilon) = s(t_1) + \epsilon\, \dot{s}(t_1) + k'(0)\, \epsilon\, \eta(t_1) \quad [11]$$

$$\dot{\bar{s}}_1 = \left(\frac{d\bar{s}}{dt}\right)_{t = t_1 + \epsilon} = \dot{\bar{s}}(t_1 + \epsilon) = \dot{s}(t_1) + \epsilon\, \ddot{s}(t_1) + k'(0)\, \epsilon\, \dot{\eta}(t_1) \quad [12]$$

where $\dot{\eta} = \frac{d\eta}{dt}$. However $\bar{s}_1$ and $\dot{\bar{s}}_1$ must satisfy Equation [5] so that the neighboring solutions will represent rockets reaching the specified summit altitude, the stated auxiliary condition. Therefore,

$$\left.\begin{aligned} \dot{\bar{s}}_1 &= \phi(s_1) + \left(\frac{d\phi}{ds}\right)_{s_1} \{\bar{s}_1 - s_1\} + \dots \\ &= \dot{s}(t_1) + \left(\frac{d\phi}{ds}\right)_{s_1} \{\bar{s}_1 - s_1\} + \dots \end{aligned}\right\} \quad [13]$$

$\bar{s}_1$ and $\dot{\bar{s}}_1$ from Equations [11] and [12] can be substituted into Equation [13]. After some simplification, one has

$$\left[\left(\frac{d\phi}{ds}\right)_{s_1} \eta(t_1) - \dot{\eta}(t_1)\right] k'(0) = \ddot{s}(t_1) - \left(\frac{d\phi}{ds}\right)_{s_1} \dot{s}(t_1) \quad [14]$$

This equation then determines the value of $k'(0)$. With the $k'(0)$ so determined, the neighboring solutions will definitely satisfy all the auxiliary conditions.

With $\eta(t)$ specified, the total initial mass M_0 will be dependent upon ϵ. Let

$$F(s, \dot{s}, t) = D(s, \dot{s}) \exp\left(\frac{\dot{s} + gt}{c}\right) \quad [15]$$

Then by substituting Equations [9] and [12] into [4],

$$M_0(\epsilon) = \frac{1}{c} \int_0^{t_1 + \epsilon} F\{s + k(\epsilon)\eta,\ \dot{s} + k(\epsilon)\dot{\eta},\ t\}\, dt +$$
$$M_1 \exp\left\{\frac{\dot{s}(t_1) + \epsilon\, \ddot{s}(t_1) + k(\epsilon)\dot{\eta}(t_1) + gt_1 + g\epsilon}{c}\right\} \quad [16]$$

The condition for $s(t)$ to correspond to the optimum solution can now be stated simply as

$$\left(\frac{\partial M_0}{\partial \epsilon}\right)_{\epsilon = 0} = 0 \quad [17]$$

By carrying out the required differentiation, one has

$$\left(\frac{\partial M_0}{\partial \epsilon}\right)_{\epsilon = 0} = \frac{1}{c} k'(0) \int_0^{t_1} \eta \left[\frac{\partial F}{\partial s} - \frac{d}{dt}\left(\frac{\partial F}{\partial \dot{s}}\right)\right] dt +$$
$$\frac{1}{c} k'(0)\, \eta(t_1) \left(\frac{\partial F}{\partial \dot{s}}\right)_{t_1} + \frac{1}{c} F(s_1, \dot{s}_1, t_1) +$$
$$\frac{1}{c} M_1 [\ddot{s}_1 + g + k'(0)\dot{\eta}(t_1)] \exp\left(\frac{\dot{s}_1 + gt_1}{c}\right)$$

But $\eta(t)$ is arbitrary other than the condition $\eta(0) = 0$. Therefore, in order for the above expression to vanish,

$$\frac{\partial F}{\partial s} - \frac{d}{dt}\left(\frac{\partial F}{\partial \dot{s}}\right) = 0 \quad [18]$$

and

$$k'(0)\eta(t_1) \left(\frac{\partial F}{\partial \dot{s}}\right)_{t_1} + F(s_1, \dot{s}_1, t_1) +$$
$$M_1 [\ddot{s}_1 + g + k'(0)\dot{\eta}(t_1)] \exp\left(\frac{\dot{s}_1 + gt_1}{c}\right) = 0 \quad [19]$$

Equation [18] is the familiar Euler-Lagrange differential equation. Equation [19] is the result of the auxiliary condition of the problem.

By eliminating $k'(0)$ between Equations [14] and [19], one has

$$0 = \left[\ddot{s}_1 - \left(\frac{d\phi}{ds}\right)_{s_1} \dot{s}_1\right] \eta(t_1) \left(\frac{\partial F}{\partial \dot{s}}\right)_{t_1} + \left[\left(\frac{d\phi}{ds}\right)_{s_1} \eta(t_1) - \dot{\eta}(t_1)\right]$$
$$\cdot F(s_1, \dot{s}_1, t_1) + M_1 \left[\left\{\left(\frac{\partial \phi}{\partial s}\right)_{s_1} \eta(t_1) - \dot{\eta}(t_1)\right\} (\ddot{s}_1 + g) + \dot{\eta}(t_1)\right.$$
$$\left. \cdot \left\{\ddot{s}_1 - \left(\frac{d\phi}{ds}\right)_{s_1} \dot{s}_1\right\}\right] \exp\left(\frac{\dot{s}_1 + gt_1}{c}\right)$$

But $\eta(t)$ is arbitrary. Therefore, in order for the above equation to be true, the sums of quantities which multiply into $\eta(t_1)$ and $\dot{\eta}(t_1)$ must be zero separately. Therefore

$$\left\{\ddot{s}_1 - \left(\frac{d\phi}{ds}\right)_{s_1} \dot{s}_1\right\} \left(\frac{\partial F}{\partial \dot{s}}\right)_{t_1} + \left(\frac{d\phi}{ds}\right)_{s_1} F(s_1, \dot{s}_1, t_1) +$$
$$M_1 \left(\frac{d\phi}{ds}\right)_{s_1} (\ddot{s}_1 + g) \exp\left(\frac{\dot{s}_1 + gt_1}{c}\right) = 0 \ldots [20]$$

and

$$F(s_1, \dot{s}_1, t_1) + M_1 \left[g + \left(\frac{d\phi}{ds}\right)_{s_1} \dot{s}_1\right] \exp\left(\frac{\dot{s}_1 + gt_1}{c}\right) = 0 \ldots [21]$$

Equations [18], [20], and [21] are now the complete answer to the variational problem. That is, the thrust programming must be such that Equation [18] is satisfied at every instant of the powered flight and, in addition, at the end of the powered flight the conditions given by Equations [20] and [21] must be fulfilled.

These conditions can be reduced to simpler forms if the relation in Equation [15] is reintroduced. Then Equation [18] becomes

$$\frac{\partial D}{\partial s} = \frac{\partial^2 D}{\partial s \partial \dot{s}} \dot{s} + \frac{\partial^2 D}{\partial \dot{s}^2} \ddot{s} +$$
$$\frac{1}{c} \left\{\frac{\partial D}{\partial s} \dot{s} + \frac{\partial D}{\partial \dot{s}} (2\ddot{s} + g) + \frac{D}{c} (\ddot{s} + g)\right\} \ldots [22]$$

When the drag is specified as a function of s and $\dot{s}$, Equation [22] gives the differential equation for the trajectory $s(t)$. Equations [20] and [21] now become

$$\left\{\ddot{s}_1 - \left(\frac{d\phi}{ds}\right)_{s_1} \dot{s}_1\right\} \left\{\left(\frac{\partial D}{\partial \dot{s}}\right)_{s_1} + \frac{D(s_1, \dot{s}_1)}{c}\right\} +$$
$$\left(\frac{d\phi}{ds}\right)_{s_1} D(s_1, \dot{s}_1) + M_1 \left(\frac{d\phi}{ds}\right)_{s_1} (\ddot{s}_1 + g) = 0 \ldots [23]$$

and

$$D(s_1, \dot{s}_1) + M_1 \left[\left(\frac{d\phi}{ds}\right)_{s_1} \dot{s}_1 + g\right] = 0 \ldots\ldots\ldots [24]$$

where the subscript $(\ )_1$ denotes the quantities evaluated at $t = t_1$. Equation [24] is, however, automatically satisfied if Equation [5] represents the relation at the beginning of coasting flight. The reason is as follows: During coasting, the rocket motor is stopped, and there is no expenditure of propellant; therefore, $dM/dt = 0$. Then Equation [1] reduces to

$$\left[\left(\frac{d\phi}{dt}\right)_{t_1} + g\right] M_1 + D(s_1, \dot{s}_1) = 0 \ldots\ldots\ldots [25]$$

This is the same as Equation [24].

By eliminating $\left(\dfrac{d\phi}{ds}\right)_{s_1}$ between Equations [23] and [24], the condition at the end of the powered flight is finally expressed as

$$\dot{s}_1(M_1\ddot{s}_1 + D_1 + M_1 g) \left\{\left(\frac{\partial D}{\partial \dot{s}}\right)_1 + \frac{D_1}{c}\right\} =$$
$$(M_1\ddot{s}_1 + D_1 + M_1 g)(D_1 + M_1 g)$$

where $D_1 = D(s_1, \dot{s}_1)$ is the drag at the end of burning. However, the factor $(M_1\ddot{s}_1 + D_1 + M_1 g)$ is never zero, therefore,

$$\dot{s}_1 \left\{\left(\frac{\partial D}{\partial \dot{s}}\right)_1 + \frac{D_1}{c}\right\} = D_1 + M_1 g \ldots\ldots\ldots\ldots [26]$$

The problem of optimum thrust programming can now be discussed in more concrete terms. Since the aerodynamic drag D enters linearly and homogeneously into Equation [22], that equation is actually a second-order differential equation for $s(t)$, independent of the size of the rocket body. However, being a second-order differential equation with only one initial condition $s(0) = 0$, the initial velocity or velocity after boosting $\dot{s}_0$ is yet free and undetermined. It is determined, however, by the condition, Equation [26], at the end of the powered flight. In other words, for a given size of the rocket and for a given final mass M_1, there is a corresponding optimum booster velocity $\dot{s}_0$ and subsequent optimum thrust programming for any specified summit altitude S. In general then, the optimum solution always involves an impulsive start from rest. This is a characteristic of the problem.

Quadratic Drag Law

To carry out the computation, the air density is assumed to decrease exponentially with respect to altitude. As a first example, the drag is taken to vary as the square of the velocity. Then the aerodynamic drag is given by

$$D = W_0 \dot{s}^2 \exp(-\alpha s) \ldots\ldots\ldots\ldots\ldots [27]$$

This corresponds to a constant drag coefficient. Substituting this expression into Equation [22] yields

$$\frac{\ddot{s}}{g} = \frac{v\{v^2 + (1 - \beta)v - 2\beta\}}{\beta\{v^2 + 4v + 2\}} \ldots\ldots\ldots\ldots [28]$$

where

$$v = \frac{\dot{s}}{c}, \qquad \beta = \frac{g}{\alpha c^2} \ldots\ldots\ldots\ldots\ldots\ldots [29]$$

both are nondimensional parameters. Integrating Equation [28] for $t(v)$ and $s(v)$,

$$\frac{gt}{c} = \ln \frac{v_0}{v} + \frac{\gamma}{2} \ln \frac{2v + (1 - \beta) - \gamma}{2v + (1 - \beta) + \gamma} \cdot \frac{2v_0 + (1 - \beta) + \gamma}{2v_0 + (1 - \beta) - \gamma} +$$
$$\frac{\beta + 1}{2} \ln \frac{v^2 + (1 - \beta)v - 2\beta}{v_0^2 + (1 - \beta)v_0 - 2\beta} \ldots [30]$$

$$\alpha s = v - v_0 +$$
$$\frac{\gamma}{2} \ln \frac{2v + (1 - \beta) - \gamma}{2v + (1 - \beta) + \gamma} \cdot \frac{2v_0 + (1 - \beta) + \gamma}{2v_0 + (1 - \beta) - \gamma} +$$
$$\frac{\beta + 1}{2} \ln \frac{v^2 + (1 - \beta)v - 2\beta}{v_0^2 + (1 - \beta)v_0 - 2\beta} \ldots [31]$$

where

$$\gamma = \sqrt{(1 - \beta)^2 + 4\beta} \ldots\ldots\ldots\ldots\ldots [32]$$

and ln indicates the natural logarithm to the base e.

By using Equations [27], [28], [30], and [31], the mass M at any time instant is found to be as follows:

$$\frac{M}{M_1} = \exp \Big\{ - \left(v + \frac{gt}{c}\right) \Big\} \Big[\frac{W_0c^2}{M_1g} \cdot \beta \cdot v_0 \cdot \exp(v_0) \times \{v_0^2 + (1-\beta)v_0 - 2\beta\} \Big\{ \frac{v+2}{v^2 + (1-\beta)v - 2\beta} - \frac{v_1 + 2}{v_1^2 + (1-\beta)v_1 - 2\beta} \Big\} + \exp\left(v_1 + \frac{gt_1}{c}\right) \Big] \ldots [33]$$

Setting $v = v_0$, $t = 0$ gives the mass M^0 after boosting:

$$\frac{M^0}{M_1} = \frac{W_0c^2}{M_1g} \cdot \beta \cdot v_0 \left[(v_0 + 2) - (v_1 + 2) \frac{v_0^2 + (1-\beta)v_0 - 2\beta}{v_1^2 + (1-\beta)v_1 - 2\beta} \right] + \exp\left(v_1 - v_0 + \frac{gt_1}{c}\right) \ldots [34]$$

The initial mass of the rocket including the booster change is given by Equation [3], or in terms of the nondimensional velocity v,

$$M_0 = M^0 \exp(v_0)$$

Therefore,

$$\frac{M_0}{M_1} = \frac{M^0}{M_1} \exp(v_0) \ldots\ldots\ldots\ldots [35]$$

The thrust at any instant can be obtained from Equation [1] noting that $F = c\,\dfrac{dM}{dt}$

$$\frac{F}{M_1g} = \frac{W_0c^2}{M_1g} v^2 \exp(-\alpha s) + \frac{M}{M_1}\left(1 + \frac{\ddot{s}}{g}\right) \ldots\ldots [36]$$

where the acceleration $\ddot{s}/g$ can be computed from Equation [28].

The end conditions are given by Equations [5] and [26]. Substituting Equation [27] into Equation [26] gives one condition

$$\frac{W_0c^2}{M_1g} v_1^2 \exp(-\alpha s_1) = \frac{1}{1 + v_1} \ldots\ldots\ldots [37]$$

After the fuel is exhausted, the equation of motion is

$$(\ddot{s} + g)M_1 + W_0\dot{s}^2 \exp(-\alpha s) = 0 \ldots\ldots\ldots [38]$$

This equation can be easily integrated. By using the condition that at the summit, $s = S$, $\dot{s} = 0$, one has the following equation for the velocity v_1 at the beginning of the coasting flight

$$v_1^2 = -2\beta \exp\left(2\beta \frac{W_0c^2}{M_1g} \xi_1\right) \int_{\xi_1}^{\xi_2} \exp\left(-2\beta \frac{W_0c^2}{M_1g} x\right) \frac{dx}{x} \ldots [39]$$

where

$$\xi_1 = \exp(-\alpha s_1), \qquad \xi_2 = \exp(-\alpha S) \ldots\ldots\ldots [40]$$

The integral in Equation [39] can be evaluated using the series expansion

$$\int \exp(-\alpha x) \frac{dx}{x} = \ln|x| - \frac{\alpha x}{1\,.\,.\,1!} + \frac{(\alpha x)^2}{2\,.\,.\,2!} - \ldots \ldots [41]$$

With any fixed value of S and the drag parameter W_0c^2/M_1g, Equations [37] and [39] determine v_1 and αs_1. The parameter W_0c^2/M_1g is the nondimensional drag and weight ratio of the rocket.

Calculations were carried out for two sets of summit altitudes and exhaust velocities assuming $\alpha = 1/22{,}000$ ft. One case was for a summit altitude of 500,000 ft and an exhaust velocity of 5500 fps. The other case was for a summit altitude of 3,000,000 ft and an exhaust velocity of 8000 fps. Actually, an iteration procedure was adopted to fit the end conditions. For any chosen value of the drag weight ratio W_0c^2/M_1g, a value of v_1 was first assumed and substituted in Equation [37] to solve for αs_1. This result was put into Equation [39] which was solved for v_1. This process was repeated until the desired accuracy was obtained. A plot of W_0c^2/M_1g versus v_1, which was found to be almost a straight line on semilogarithmic graph paper, with v_1 plotted on the linear scale, simplified the process by giving an excellent first estimate of the velocity. The initial velocity ratio v_0 was then solved for by a trial and error procedure using Equation [31].

Values of W_0c^2/M_1g were assumed for the calculations, but the final results were put in terms of M_0. This was done because physically the results in terms of the initial mass are easier to visualize. The quantity W_0c^2 is the drag of the rocket body at a flight velocity equal to the exhaust velocity c and at sea level. The parameter then expresses the ratio of this drag to initial weight. The results are shown in Figs. 1 to 8. The discussion of these results is postponed until a later section.

Linear Drag Law

For supersonic flight of a rocket, perhaps, a better approximation of the drag is given by

$$D = (A\dot{s} + B) \exp(-\alpha s) \ldots\ldots\ldots\ldots [42]$$

This again corresponds to an exponential air-density law, but the drag coefficient decreases as the velocity increases. Substituting Equation [42] into Equation [22] gives

$$\frac{\ddot{s}}{g} = \frac{v^2 + (B/Ac - \beta)v - [\beta + B/Ac(1+\beta)]}{\beta(v + 2 + B/Ac)} \ldots [43]$$

where v and β are the same as the previous section and are defined by Equation [29]. Equation [43] can be integrated and gives

$$\frac{gt}{c} = \frac{\beta}{2} \ln \frac{v^2 + v(B/Ac - \beta) - [\beta + B/Ac(1+\beta)]}{v_0^2 + v_0(B/Ac - \beta) - [\beta + B/Ac(1+\beta)]} + \frac{\beta(4 + \beta + B/Ac)}{2\lambda} \ln \frac{2v + (B/Ac - \beta) - \lambda}{2v + (B/Ac - \beta) + \lambda} \cdot \frac{2v_0 + (B/Ac - \beta) + \lambda}{2v_0 + (B/Ac - \beta) - \lambda} \ldots [44]$$

and

$$\alpha s = v - v_0 + \left(1 + \frac{\beta}{2}\right) \ln \frac{v^2 + v(B/Ac - \beta) - [\beta + B/Ac(1+\beta)]}{v_0^2 + v_0(B/Ac - \beta) - [\beta + B/Ac(1+\beta)]} + \frac{\beta(4 + \beta + B/Ac)}{2\lambda} \ln \frac{2v + (B/Ac - \beta) - \lambda}{2v + (B/Ac - \beta) + \lambda} \cdot \frac{2v_0 + (B/Ac - \beta) + \lambda}{2v_0 + (B/Ac - \beta) - \lambda} \ldots [45]$$

where

$$\lambda = \sqrt{(B/Ac + \beta)^2 + 4(B/Ac + \beta)} \ldots\ldots\ldots [46]$$

The mass M at any instant is found to be

$$\frac{M}{M_1} =$$

$$\left[\frac{Ac}{M_1 g}\cdot\beta(\exp v_0)\,\{v_0^2 + v_0(B/Ac - \beta) - (\beta + B/Ac[1+\beta])\}\,\times\right.$$
$$\left\{\frac{v + B/Ac + 1}{v^2 + v(B/Ac - \beta) - (\beta + B/Ac[1+\beta])} - \frac{v_1 + B/Ac + 1}{v_1^2 + v_1(B/Ac - \beta) - (\beta + B/Ac\,[1+\beta])}\right\} +$$
$$\left.\exp\left(v_1 + \frac{gt_1}{c}\right)\right]\exp\left\{-\left(v + \frac{gt}{c}\right)\right\}\ldots[47]$$

The initial mass M_0 including the boosting can be obtained from the above equation and Equation [35]

$$\frac{M_0}{M_1} = \exp\left(v_1 + \frac{gt_1}{c}\right) +$$
$$\frac{Ac}{M_1 g}\,\beta\left[\{v_0 + B/Ac + 1\} - \{v_1 + B/Ac + 1\}\,\times\right.$$
$$\left.\frac{v_0^2 + v_0(B/Ac - \beta) - (\beta + B/Ac[1+\beta])}{v_1^2 + v_1(B/Ac - \beta) - (\beta + B/Ac[1+\beta])}\right]\cdot\exp(v_0)\ldots[48]$$

The thrust can be obtained from Equation [1]. Thus

$$\frac{F}{M_1 g} = \frac{Ac}{M_1 g}(v + B/Ac)\exp(-\alpha s) + \frac{M}{M_1}\left(1 + \frac{\ddot{s}}{g}\right)\ldots[49]$$

The acceleration s/g can be determined from Equation [43].

The end condition is given by Equations [26] and [5]. Substituting Equations [42] and [43] into Equation [26], one has

$$\frac{Ac}{M_1 g}(v_1^2 + v_1 B/Ac - B/Ac)\exp(-\alpha s_1) = 1\ldots[50]$$

To supply the relation indicated by Equation [5], the coasting flight trajectory must be determined. By using Equation [42], the equation of motion for the coasting flight is

$$\left(\frac{\ddot{s}}{g} + 1\right) + \frac{Ac}{M_1 g}(v + B/Ac)\exp(-\alpha s) = 0\ldots\ldots[51]$$

This equation cannot be solved by simple quadrature. However, the effect of air drag is generally small. Then Equation [51] can be solved approximately by first neglecting the air drag and then making the necessary small correction. Let v^0 be the value of the velocity ratio without air drag and v' be the small correction on v. Then

$$v = v^0 + v'\ldots\ldots\ldots\ldots\ldots\ldots\ldots[52]$$

The equation for v^0 is simply

$$\frac{d(v_0)^2}{ds} + \frac{2g}{c^2} = 0\ldots\ldots\ldots\ldots\ldots\ldots[53]$$

Now substitute Equation [52] into Equation [51] and retain only linear terms in v',

$$v^0\frac{dv'}{ds} + v'\frac{dv^0}{ds} + \frac{A}{M_1 c}(v^0 + B/Ac)\exp(-\alpha s) = 0\ldots[54]$$

This is the differential equation for v', the correction term. Equation [53] has the solution

$$v^0 = \sqrt{2\beta(\alpha S - \alpha s)}\ldots\ldots\ldots\ldots\ldots[55]$$

Substitute Equation [55] into Equation [54],

$$\frac{dv'}{d\zeta} + \frac{v'}{2\zeta} = \frac{Ac}{M_1 g}\,\beta\left\{1 + \frac{B/Ac}{\sqrt{2\beta\zeta}}\right\}\exp(\zeta - \alpha S)$$

where

$$\zeta = \alpha S - \alpha s$$

Solving for v'

$$v' = \frac{1}{\sqrt{\zeta}}\,\frac{Ac}{M_1 g}\,\beta\exp(-\alpha S)\int_0^\zeta\left\{\sqrt{x} + \frac{B/Ac}{\sqrt{2\beta}}\right\}\exp(x)\,dx\,.\,[56]$$

To obtain a suitable series for the computation of the integral in Equation [56], consider

$$f(\zeta) = \int_0^\zeta\sqrt{x}\exp(x)dx$$

Let $x = \zeta(1 - u)$, then

$$f(\zeta)\exp(-\zeta) = \zeta^{3/2}\int_0^1\sqrt{1-u}\exp(-\zeta u)\,du$$

By expanding the radical in the integrand into a power series and integrating term by term, $f(\zeta)$ can be shown to be equal to the following series

$$f(\zeta) = \sqrt{\zeta}\left[\{\exp(\zeta) - 1\} - \frac{1}{2\zeta}\{\exp(\zeta) - (1+\zeta)\} - \frac{1}{4\zeta^2}\left\{\exp(\zeta) - \left(1 + \zeta + \frac{\zeta^2}{2}\right)\right\} - \ldots\right]$$

Then

$$v' = \frac{1}{\sqrt{\zeta}}\cdot\frac{Ac}{M_1 g}\,\beta\left\{f(\zeta) + B/Ac\frac{1}{\sqrt{2\beta}}(\exp(\zeta) - 1)\right\}\times\exp(-\alpha S)\ldots[57]$$

Equations [50] and [57] must be solved simultaneously to determine the velocity and altitude at the end of burning for a given drag weight ratio, Ac/M_1g, for the rocket. The quantity Ac is now the drag of the rocket body at a flight velocity equal to the exhaust velocity c and at sea level, if $B = 0$.

Calculations were carried out for a summit altitude of 3,000,000 ft, an exhaust velocity of 8000 fps, and no drag at zero velocity, assuming $\alpha = 1/22{,}000$ ft. An iteration procedure identical to that used in the previous example was used. The results are shown in Figs. 9 to 12. The parameter here is Ac/M_0g, the ratio of the drag of the rocket at velocity c and at sea level to the initial weight.

In order to determine the gain possible by using the optimum thrust programming as against the conventional constant thrust rocket, a few cases of constant thrust were computed using the same drag laws. It was assumed the constant thrust rocket had no initial velocity and the ratio of thrust to initial weight F/M_0g was 2.7. This was chosen to agree with the best initial acceleration given by Ivey, Bowen, and Oborny (2) for high-performance rockets. The comparisons were based on equal ratios of the drag at the exhaust velocity c and at sea level to the initial weight M_0g and summit altitude S. The mass ratio M_0/M_1 is now larger than the optimum value due to the fact that the final mass M_1 is now smaller. A step-by-step procedure was used in computing the mass ratio of final mass to initial mass which gave an error of approximately one per cent. This error was always nonconservative and would therefore favor the con-

stant-thrust examples in the comparisons. The results are indicated by the dots in Figs. 1, 2, and 9.

Discussion of Results

Both the linear drag law and the quadratic drag law cases have certain common characteristics. As seen from Figs. 3, 4, 7, 8, 10, and 12, for low values of the drag parameter, the average thrust and initial velocity were high. Therefore, the rocket will burn its fuel rapidly and less energy will be expended against gravity. As the drag parameter increases, more energy is used to overcome the drag, and it becomes advantageous to have a lower velocity at low altitudes. Therefore, the initial velocity and thrust will be lower, and the altitude at the end of burning will be greater than in the case of low drag parameter. But it is important to note that there is always a finite initial velocity $\dot{s}_0$, so that the optimum thrust programming involves a boosting impulse. Another result the two drag law cases have in common is the effect of the drag parameters on the ratio of the initial mass to the final mass, M_0/M_1 (Figs. 1, 2, and 9). As the drag parameters increase, this ratio M_0/M_1 increases for a given summit altitude. As the drag parameters decrease, the ratio M_0/M_1 asymptotically approaches the no-drag value of exp $\dot{s}_0/c$ where

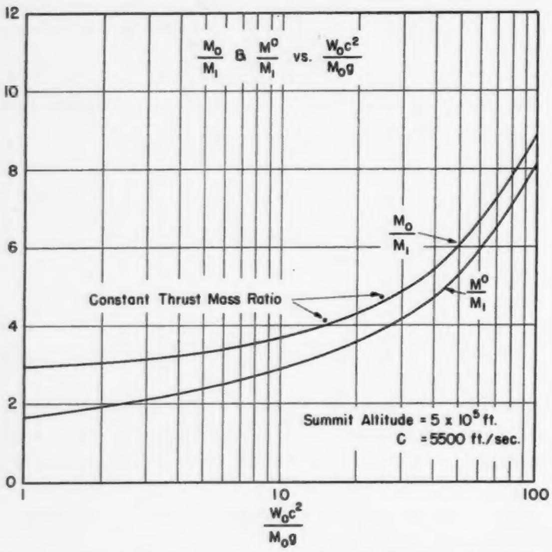

FIG. 1 INITIAL MASS RATIOS VERSUS THE DRAG RATIO W_0c^2/M_0g, THE RATIO OF THE SEA LEVEL DRAG AT A VELOCITY EQUAL TO THE EXHAUST VELOCITY c, TO THE INITIAL WEIGHT M_0g

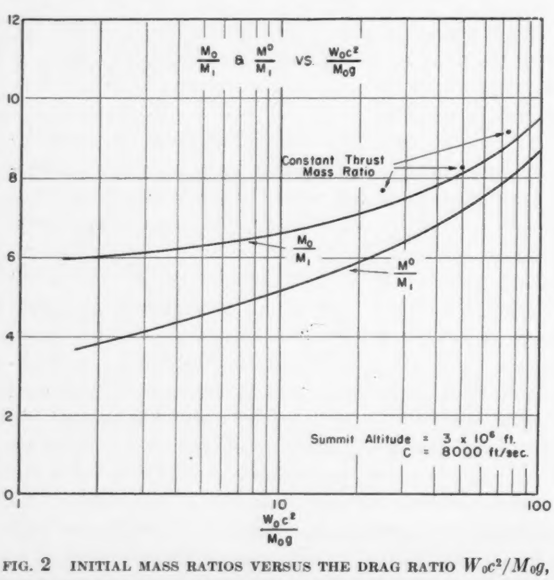

FIG. 2 INITIAL MASS RATIOS VERSUS THE DRAG RATIO W_0c^2/M_0g, THE RATIO OF THE SEA LEVEL DRAG AT A VELOCITY EQUAL TO THE EXHAUST VELOCITY c, TO THE INITIAL WEIGHT M_0g

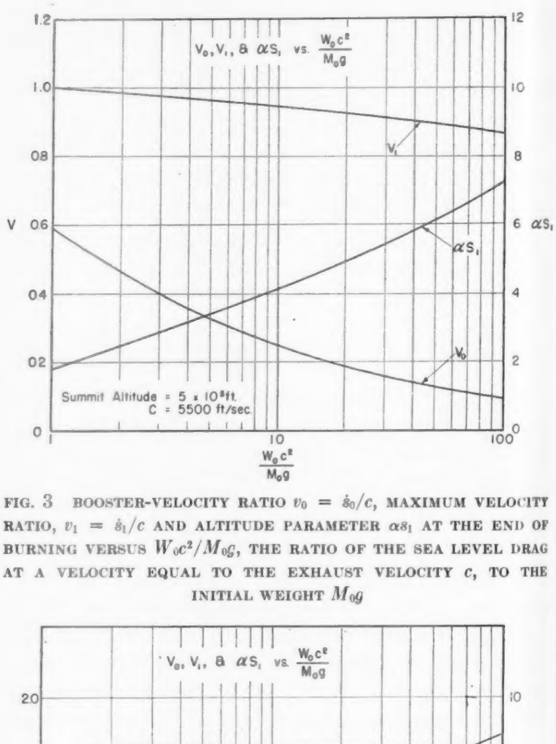

FIG. 3 BOOSTER-VELOCITY RATIO $v_0 = \dot{s}_0/c$, MAXIMUM VELOCITY RATIO, $v_1 = \dot{s}_1/c$ AND ALTITUDE PARAMETER αs_1 AT THE END OF BURNING VERSUS W_0c^2/M_0g, THE RATIO OF THE SEA LEVEL DRAG AT A VELOCITY EQUAL TO THE EXHAUST VELOCITY c, TO THE INITIAL WEIGHT M_0g

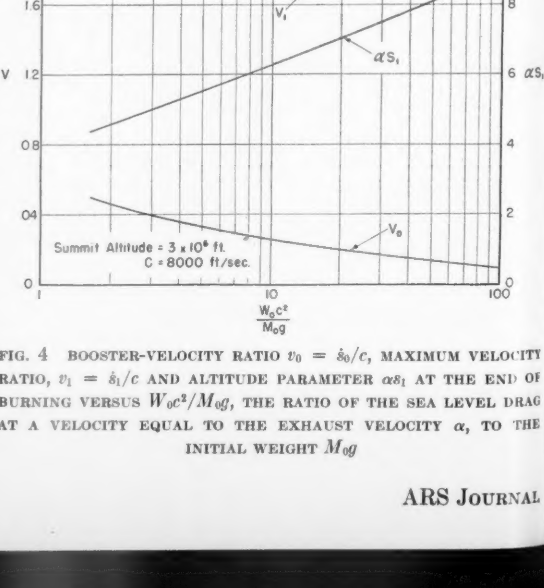

FIG. 4 BOOSTER-VELOCITY RATIO $v_0 = \dot{s}_0/c$, MAXIMUM VELOCITY RATIO, $v_1 = \dot{s}_1/c$ AND ALTITUDE PARAMETER αs_1 AT THE END OF BURNING VERSUS W_0c^2/M_0g, THE RATIO OF THE SEA LEVEL DRAG AT A VELOCITY EQUAL TO THE EXHAUST VELOCITY α, TO THE INITIAL WEIGHT M_0g

$\dot{s}_0$ is such that the rocket has the kinetic energy at zero altitude equal to the potential energy at the summit altitude.

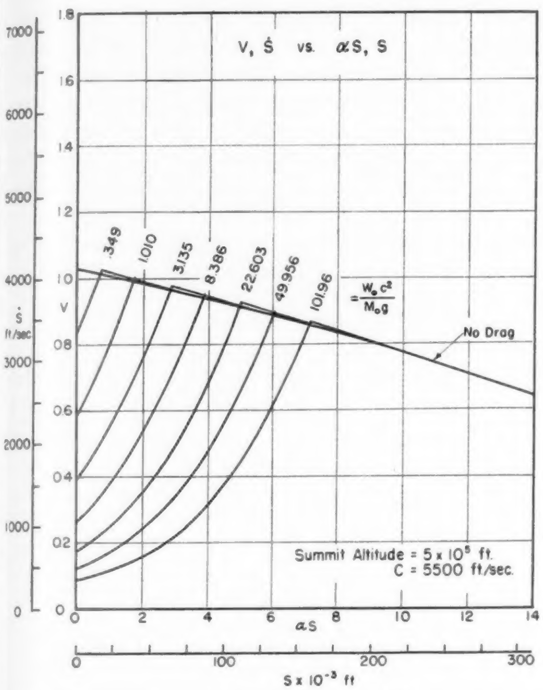

FIG. 5 VELOCITY $\dot{s}$ VERSUS ALTITUDE s, W_0c^2/M_0g IS THE DRAG RATIO

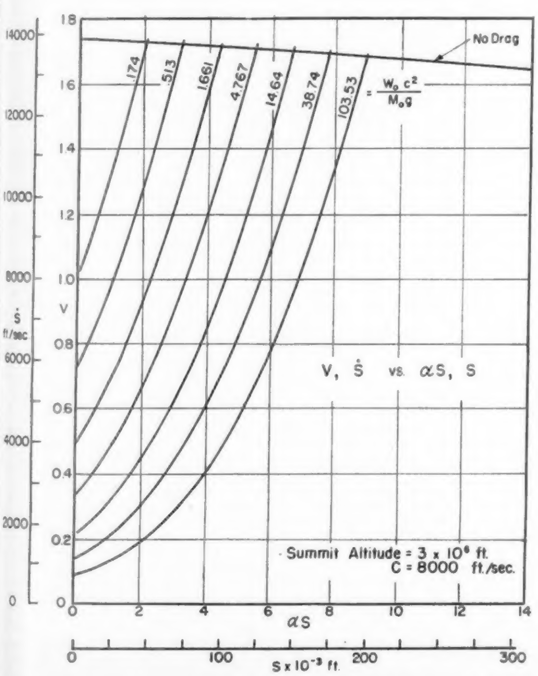

FIG. 6 VELOCITY $\dot{s}$ VERSUS ALTITUDE s, W_0c^2/M_0g IS THE DRAG RATIO

In general, for optimum programming, the thrust after the initial impulse should increase with altitude. Exceptions to this occur for portions of the powered flight when the values of the drag parameter are very high, or when the rocket has a very high performance. In these cases, for a portion of the burning time, the thrust can be seen to decrease. This is the result of either a very slowly increasing acceleration of the rocket or a very rapidly decreasing mass of the rocket. Since the accelerating force is the major fraction of the thrust and equal to the product of the instantaneous mass and acceleration, the thrust can decrease. In general, however, the thrust should increase. For a constant geometry rocket motor, this is accomplished to a limited extent by the decreasing atmospheric pressure acting on the nozzle exit. But for most cases, such an increase is only a fraction of the ideal, as determined by the present calculation.

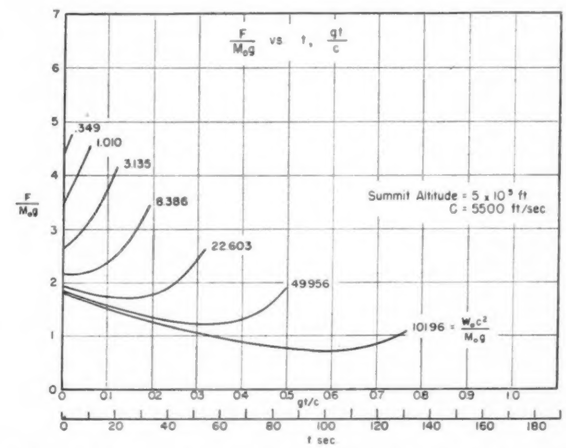

FIG. 7 F/M_0g, THE RATIO OF THE THRUST TO THE INITIAL WEIGHT VERSUS TIME t. W_0c^2/M_0g IS THE DRAG RATIO

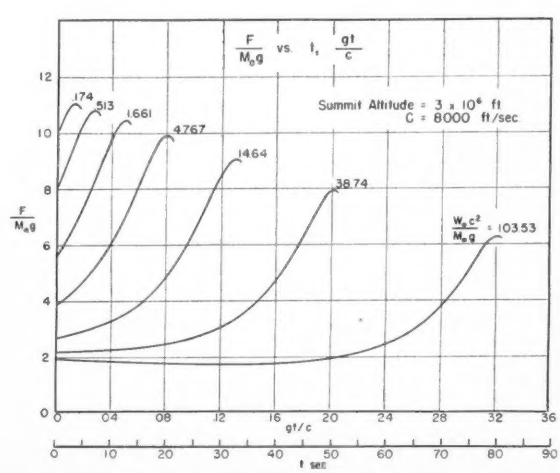

FIG. 8 F/M_0g, THE RATIO OF THE THRUST TO THE INITIAL WEIGHT VERSUS TIME t. W_0c^2/M_0g IS THE DRAG RATIO

The linear drag law case has a higher initial velocity and shorter burning time than has the quadratic drag law case for similar mass ratios M_0/M_1. This is true because the drag does not increase as rapidly with velocity in the linear drag law case; therefore, the linear drag law case will favor the higher velocities and shorter burning times. The drag in the quadratic law case increases as the square of the velocity; therefore, this case will favor the lower velocities and longer burning times.

To give an example of the magnitudes of the ratios W_0c^2/M_0g and Ac/M_0g for the drag and the initial weight, consider the case of the V-2 rocket. At a velocity of 6000 fps the drag is roughly 100,000 lb. The initial weight of the V-2 is approximately 25,000 lb. This gives a ratio of drag at the exhaust velocity at sea level to the initial weight of roughly 4 if the exhaust velocity is 6000 fps. Therefore, 4 might be an approximate value of the ratio Ac/M_0g. In the quadratic drag law case, a better estimate of W_0c^2/M_0g might be 10. This will account for the quadratic drag law giving a lower value of the drag than is actually the case during the major portion of the powered flight, especially at the lower altitudes. Since the summit altitude of the V-2 is

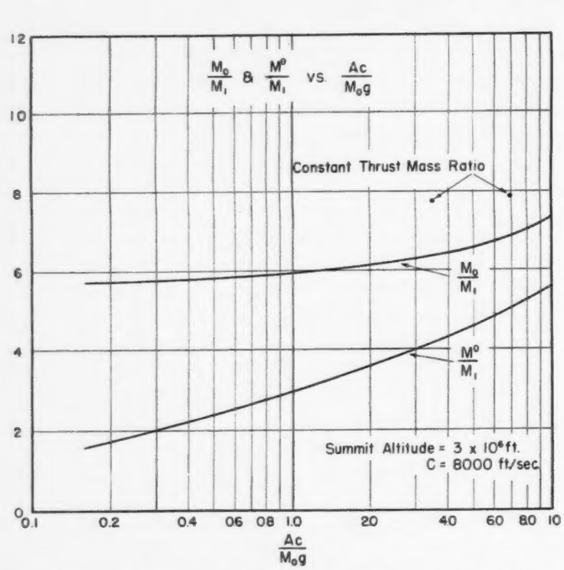

FIG. 9 INITIAL MASS RATIOS VERSUS THE DRAG RATIO Ac/M_0g, THE RATIO OF THE SEA LEVEL DRAG AT A VELOCITY EQUAL TO THE EXHAUST VELOCITY c, TO THE INITIAL WEIGHT M_0g

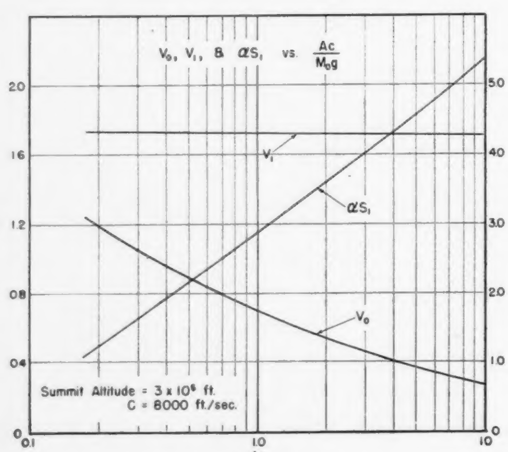

FIG. 10 BOOSTER-VELOCITY RATIO $v_0 = \dot{s}_0/c$, MAXIMUM VELOCITY RATIO $v_1 = \dot{s}_1/c$ AND ALTITUDE PARAMETER αs_1, AT THE END OF BURNING VERSUS Ac/M_0g, THE RATIO OF THE SEA LEVEL DRAG AT A VELOCITY EQUAL TO THE EXHAUST VELOCITY c, TO THE INITIAL WEIGHT M_0g

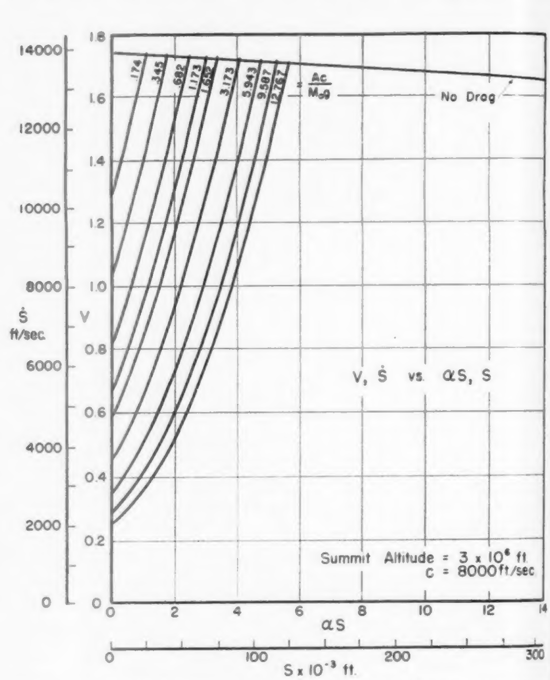

FIG. 11 VELOCITY $\dot{s}$ VERSUS ALTITUDE s. Ac/M_0g IS THE DRAG RATIO

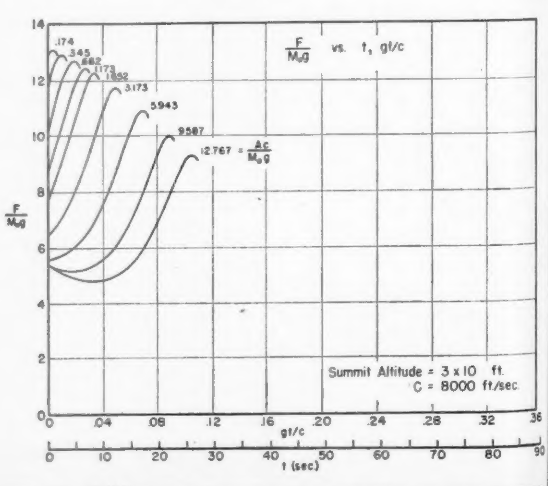

FIG. 12 F/M_0g, THE RATIO OF THE THRUST TO THE INITIAL WEIGHT VERSUS TIME t. Ac/M_0g IS THE DRAG RATIO

approximately 100 miles, Fig. 3 can be used to estimate the optimum booster velocity to be roughly 1500 fps, assuming $W_0c^2/M_0g = 10$. If the drag is relatively less, the booster velocity will increase. Therefore, the optimum thrust programming involves considerable boosting.

Several constant-thrust mass ratios were calculated for comparison with the ideal mass ratios. It was assumed the constant thrust rockets had no initial velocity and followed the same drag laws as the ideal cases. An initial acceleration of $1.7g$ was chosen to agree with the best initial acceleration given by Ivey, Bowen, and Oborny (2) for high performance rockets. This value might not be the optimum for the constant thrust cases computed because it is dependent on the drag laws followed by the rocket. The values for the constant thrust rocket calculations can be seen in Figs. 1, 2, and 9. The advantages of the ideal rocket would be less if the constant thrust rocket had the best possible initial velocity and acceleration. In fact, it seems certain that the initial acceleration of $1.7g$ is too small for the linear drag case of Fig. 9. The possible gain by using optimum thrust programming is thus more correctly represented by Fig. 2. Then it is seen that the gain is small in terms of the initial mass M_0. On the other hand, one must bear in mind that the pay load of a high altitude sounding rocket is only a very small fraction of the initial weight. Therefore, even if the gain by better thrust programming may be small in terms of the initial weight, the gain in terms of pay load could be appreciable. Of course, the numerical results obtained by the present calculation are only qualitative. For any practical design, besides consideration for the additional weight needed for thrust variation, the optimum-thrust programming has to be determined with a more factual drag versus velocity relation.

Acknowledgment

During his stay at the California Institute of Technology, Prof. Raymund Sänger of Eidgenössiche Technische Hochschule, Zürich, Switzerland, has kindly checked the mathematics of this calculation, and suggested a few important simplifications. For this, the authors are deeply grateful.

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A Theory of Unstable Combustion in Liquid Propellant Rocket Systems

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On the basis of an hypothesis that low-frequency oscillations (chugging) sometimes observed in liquid propellant rocket engines, are the result of oscillatory propellant flow induced by a combustion time lag, conditions for the suppression of such oscillations are derived. It is found that stability can be achieved by increases in the length of feed line, the velocity of the propellant in the feed line, the ratio of feed pressure to chamber pressure, and the ratio of chamber volume to nozzle area. Equations are given for the frequencies of oscillation. Examination of the equation for stability indicates that self-igniting propellant combinations are likely to be more stable than non-self-igniting systems.

Nomenclature

Physical Quantities

A = cross-sectional area (normal to the flow direction)
c^* = characteristic velocity of the propellant
K = over-all frictional pressure loss coefficient
l = length of flow passage
L^* = characteristic length of the rocket motor
$\dot{m}$ = mass flow (mean value) through the rocket system
p = pressure (absolute)
Δp = $p_1 - p_c$ = pressure difference from tank to chamber
R = gas constant of combustion gas (based on unit mass)
T = temperature (absolute)
v = velocity of flow
V = volume of chamber
β = volume compressibility of liquid
λ = damping factor of oscillation
ω = frequency of flow oscillation (radians/sec)
ρ = density of the liquid
τ = combustion time lag

Subscripts

$_1$ refers to propellant tank
$_2$ refers to feed line
$_3$ refers to injector orifice
$_c$ refers to combustion chamber
$_t$ refers to exhaust nozzle throat
$_e$ refers to equilibrium value of v
$_g$ refers to combustion gas

Parameters

A, B, C, a, b, c, U, u, x, θ are employed in the analysis to simplify algebraic manipulation and are defined in the text.

Introduction

Liquid propellant rocket engines of all types are generally designed by the engineer to deliver a fairly constant steady thrust for a duration that may extend from several seconds to as much as an hour. However, in many cases it has been reported that the desired steady operation does not occur in actual test, and instead a condition variously described as "rough burning," "chugging," "screaming," or simply unstable combustion may take place. Frequencies ranging from 10 cycles per sec to as much as 5000 cycles per sec have been observed in oscillographic chamber pressure traces, amplitudes from a few per cent to as much as 50 per cent of the mean chamber pressure, and in many cases, the oscillation was not truly periodic but seemed to be merely a series of random fluctuations. Not only is the resulting thrust vibration undesirable from the standpoint of possible damage to the structural elements or instruments in the vehicle, but in extreme cases, failure of the power plant itself can occur (1).[1]

The theory presented here does not attempt to explain all cases of unstable combustion, but it may provide an explanation for one type of instability, perhaps the most serious type, sometimes called chugging.

The phenomenon of chugging was first observed by the author in October, 1941, during a series of tests of a 1000-lb-thrust, nitric-acid–gasoline rocket motor at the Jet Propulsion Laboratory of the California Institute of Technology. Upon ignition, combustion would proceed smoothly at first, but would rapidly become rough, and in many instances, after five or ten seconds, severe fluctuations in chamber pressure would be taking place. Frequently, the bolts of the combustion chamber would rupture before the propellant flow could be turned off.

An explanation for this oscillation was advanced on the basis of an hypothesis that a time lag existed between an arbitrary fluctuation in propellant flow and its subsequent manifestation in the combustion chamber pressure. This time lag is the result of the finite rate of the over-all combustion process, and is determined, therefore, by the kinetic rates of mixing, vaporization, and chemical reaction. The argument was pursued to show that the instability could be suppressed by increasing the pressure drop across the injector. A numerical illustration of this argument may be helpful for an appreciation of the analysis below.

A particular rocket engine with a compressed-gas type of feed system is designed to operate with a chamber pressure of 300 psia.. In the first case, assume that the feed pressure is 500 psia. Now suppose a momentary decrease in chamber pressure occurs after steady

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[1] Numbers in parentheses refer to references on page 114.

operation is achieved, the decrease being from 300 to 200 psia. Assuming a square-law pressure drop across the injector, and neglecting the inertia of the liquid in the feed line, the flow rate will increase by 23 per cent above the design value and will subsequently (after the time lag mentioned above) produce a 23 per cent increase in chamber pressure above the design value, namely, 369 psia. The injector pressure drop is now only 131 psia, the corresponding flow rate is therefore reduced to 81 per cent of the design value, and the chamber pressure will then fall (after the time lag), to 81 per cent or 243 psia. The flow rate then increases again, and it is possible to continue the calculation in the same manner. In this case, after one cycle, it is apparent that the disturbance is decaying, the amplitude having decreased from 100 psia to 69 psia to 57 psia.

Next, consider a second case in which the design chamber pressure is also 300 psia, but the feed pressure is only 400 psia. If an arbitrary decrease of 100 psia is assumed as above, the sequence of chamber pressure values becomes 300, 200, 420, 0. In this case it is clear that the fluctuations will increase in strength as time proceeds. A third case, assuming a feed pressure of 450 psia, exhibits neutral stability, i.e., the amplitude of the fluctuation remains approximately constant.

It appears from such crude considerations that the injector pressure drop is a controlling parameter. It will be seen below that this numerical conclusion can be generalized in the following statement: Instability is not possible if the injector pressure drop exceeds one half the mean chamber pressure. However, it will be seen that the converse is not always true. That is, instability can be suppressed even when the injector drop is less than half the chamber pressure.

Of course, even in its qualitative form, the theory suggests that instability can be eliminated by reducing the time lag in the combustion process. This was accomplished, in the 1941 program, by switching to aniline as a fuel instead of gasoline, and indeed, with no change in design or operating conditions, smooth running was achieved.

In formulating the problem in 1941, the author's analysis neglected the inertia of the liquid propellant in the feed lines.[2] The flexible coupling responsible for this self-excited oscillation was assumed to be the capacitance of the combustion chamber. A recent paper by Gunder and Friant (2) presented an analysis of chugging in which the essential hypothesis was also the time lag described above (it was independently conceived by them), but instead of the chamber capacitance the important term was the inertia of the liquid in the feed lines. It is shown in their paper that instability is not possible when the pressure drop exceeds half the chamber pressure. Yachter and Waldinger, in an unpublished communication, 1949, considered the case in which liquid inertia is neglected, and reached the same conclusion.

Since similar conclusions resulted either from a consideration of liquid inertia alone or chamber capacitance alone, it appeared logical to carry out the analysis with both effects present. This is done below, and not only does it include each of the above analyses as special cases, but certain conditions for stability can be derived in such simple form that physical conclusions suggest themselves immediately.

In passing, a few remarks concerning other types of nonsteady operation are pertinent in order to clarify the assumptions of the present theory. It is assumed here that the pressure is uniform throughout the combustion chamber at any instant, that is, that the period of oscillation is long compared with the time required for a pressure wave to traverse the combustion chamber in any direction. However, high-frequency screaming has been observed in some rocket motor tests, which indicates that chamber gas oscillations are possible, in addition to the system oscillations treated here. These high frequencies usually agree fairly well with those expected on the basis of axial resonance (organ pipe notes) or transverse resonance. What is not clear, however, is the nature of the process that energizes such oscillations and why they appear only under certain conditions of injection or chamber configuration. It is suspected that the oscillation is energized by a proper interaction between the variations in pressure and the rate of combustion (that is, the time lag is some function of the instantaneous pressure), but the exact formulation of this interaction is not known.[3] The remedy is not clear unless it is to try arbitrary changes in the spatial combustion pattern. These high-frequency vibrations are usually of small amplitude but are generally accompanied by significant increases in heat transfer to the chamber walls. The danger of this type of oscillation lies in the possibility of a burn-out of the chamber.

A third category of unsteady operation is connected with either mechanical or hydraulic oscillations in the rocket system. Aside from the well-known possibilities of structural vibration, valve flutter, feed pump pulsation, etc., there seems to be the additional possibility of nonsteady hydraulic behavior of certain elements of the flow system, particularly the injector orifices. The danger of operating near the critical flow rate corresponding to the appearance of a vena-contracta in the injector orifice was first pointed out to the author by W. B. Powell[4] in 1942. The remedy

[2] The author is indebted to Dr. Theodore von Kármán for profitable discussion that led to the basic hypothesis of a combustion time lag and for suggestions on the mathematical approach to the problem. (Private communication, November, 1941.)

[3] An analysis based on this hypothesis has been advanced by Dr. L. Crocco of Princeton University.

[4] Jet Propulsion Laboratory, California Institute of Technology.

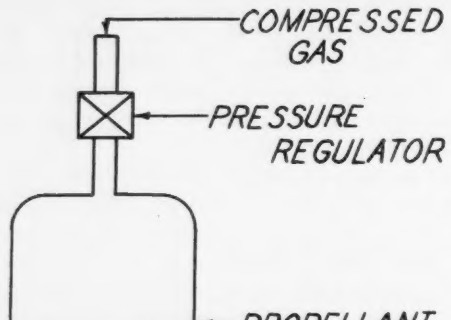

FIG. 1. SCHEMATIC LIQUID PROPELLANT ROCKET SYSTEM

for this type of instability lies clearly within the domain of the hydraulic engineer. The present theory is not concerned with such effects.

Analysis

A schematic liquid propellant rocket system is shown in Fig. 1. It is assumed that the system is either a monopropellant type or, if a bi- or multipropellant system is under consideration, that the feed lines and injector orifices for the separate liquid reactants have identical hydraulic characteristics so that it behaves like a monopropellant system. Although it is possible to repeat the monopropellant analysis (below) for the case of a multipropellant system, the additional complication is hardly worth while until the basic ideas are confirmed by experimental checks. The physical principles and the resulting rules for overcoming instability will be qualitatively the same for the bipropellant case.

At any instant during the oscillation the rate of change of pressure in the combustion chamber is governed by the difference between the rate of evolution of combustion gas and the rate of gas flow through the nozzle.[5] In accordance with the fundamental hypothesis of this theory, the rate of evolution of gas at a given instant is equal to the rate of flow of propellant *at an instant τ sec earlier.*

$$\frac{dp_c}{dt} = \frac{RT_c}{V_c}\rho A_2 v_2(t-\tau) - \frac{RT_c}{L^* c^*} p_c \qquad [1]$$

The effect of a changing chamber pressure on the flow of liquid propellant can be calculated by first setting up the general equation for the instantaneous flow rate with a specified difference between tank pressure and chamber pressure, and then differentiating this equation with respect to time. A convenient approach is to consider two control surfaces normal to the flow direction, one of area A_1 in the propellant tank upstream of the inlet to the feed line, and the other of area A_3 at the exit of the injector orifice, and then equate the total energy (work plus kinetic energy) entering the first control surface in a time interval dt to the energy leaving the second control surface plus the change of kinetic energy of the liquid contained between the two control surfaces plus the energy dissipated in the same time interval.

$$\left(p_1 A_1 v_1 + \frac{1}{2}\rho A_1 v_1^3\right) dt = \left(p_c A_3 v_3 + \frac{1}{2}\rho A_3 v_3^3\right) dt + d\left(\frac{1}{2} A_1 l_1 \rho v_1^2 + \frac{1}{2} A_2 l_2 \rho v_2^2 + \frac{1}{2} A_3 l_3 \rho v_3^2\right) + \rho\frac{1}{2}\left(k_1 v_1^2 + k_2 v_2^2 + k_3 v_3^2\right) dt$$

Carrying out the differentiation, replacing v_1 and v_3 by v_2 through the continuity equation, the equation takes the following form:

$$(p_1 - p_c) + \frac{1}{2}\rho v_2^2\left[\left(\frac{A_2^2}{A_1^2} - \frac{A_2^2}{A_3^2}\right) - K\right] = \rho\left(l_1\frac{A_2}{A_1} + l_2 + l_3\frac{A_2}{A_3}\right)\frac{dv_2}{dt} \qquad [2]$$

In practical designs, $\frac{A_2}{A_1} \ll \frac{A_2}{A_3}$, $l_1\frac{A_2}{A_1} \ll l_2$, and $l_3\frac{A_2}{A_3} \ll l_2$. Hence, without appreciable error, these terms may be neglected. The energy dissipations due to pipe line friction, valve losses, and imperfect flow in the injector orifices are contained in the term $K\frac{1}{2}\rho v_2^2$.

Differentiating Equation [2] with respect to time, the desired equation is

$$\frac{dp_c}{dt} + \rho\left(\frac{A_2^2}{A_3^2} + K\right) v_2\frac{dv_2}{dt} + \rho l_2\frac{d^2 v_2}{dt^2} = 0 \qquad [3]$$

Equations [1] and [3] can be combined to eliminate p_c and provide a single equation with a single dependent variable, v_2. Thus, differentiate Equation [1] to produce an equation containing the first and second derivative of p_c. The first derivative can be replaced by Equation [3], and an expression for the second derivative is obtainable by differentiating Equation [3]. The resulting equation for v_2 is the following (dropping

[5] Symbols employed in the analysis are explained on page 108.

subscript 2 of v_2, and writing prime for the derivative with respect to time):

$$l_2 v''' + \frac{RT_c l_2}{c^* L^*} v'' + \left(\frac{A_2^2}{A_3^2} + K\right) v v'' + \left(\frac{A_2^2}{A_3^2} + K\right) (v')^2 + \frac{RT_c}{c^* L^*} \left(\frac{A_2^2}{A_3^2} + K\right) v v' + \frac{A_2 RT_c}{V_c} v'_{(t - \tau)} = 0 \quad [4]$$

This nonlinear differential equation can be linearized by the following argument. If the amplitude of oscillation Δv is small compared with the equilibrium value of v (which is a plausible assumption in testing for instability), then v_e can replace v in Equation [4]. From Equation [2]

$$v_e = \left(\frac{A_2^2}{A_3^2} + K\right)^{-1/2} \left(\frac{2\Delta p}{\rho}\right)^{1/2} \quad [5]$$

The same assumption that Δv is small compared with v_e makes it legitimate to throw out the $(v')^2$ term in comparison with either the v'' terms or the v' terms. The latter comparison leads to this conclusion provided RT_c/c^*L^* is identified as the maximum value of the frequency when conditions are near neutral stability. This identification appears later and provides *a posteriori* justification for this approximation.

Making the substitution, $u = v'$, the differential equation becomes

$$u'' + Au' + Bu + Cu_{(t - \tau)} = 0 \quad [6]$$

In this equation

$$\left.\begin{aligned} A &= \frac{RT_c}{c^* L^*} + \left(\frac{A_2^2}{A_3^2} + K\right)\frac{v_e}{l_2} \\ B &= \frac{RT_c}{c^* L^*} \cdot \left(\frac{A_2^2}{A_3^2} + K\right)\frac{v_e}{l_2} \\ C &= \frac{RT_c}{V_c} \cdot \frac{A_2}{l_2} \end{aligned}\right\} \quad [7]$$

To solve Equation [6], it is assumed that the general solution can be represented in the form of a linear sum of particular solutions, each of these being a sinusoidal oscillation modified by a damping factor.[6]

$$u = \sum_n U_n e^{(\lambda_n + i\omega_n)t} \quad [8]$$

The successive values (λ_n, ω_n) are the roots of the characteristic equations obtained by inserting the above solution into Equation [6], and grouping real and imaginary terms:

$$\lambda^2 + A\lambda + B - \omega^2 + Ce^{-\lambda\tau} \cos(\omega\tau) = 0 \quad [9]$$

$$2\lambda\omega + A\omega - Ce^{-\lambda\tau} \sin(\omega\tau) = 0 \quad [10]$$

It is convenient to transform these equations by introducing new variables:

$$\left.\begin{aligned} \omega\tau &= \theta \\ \lambda\tau &= x \\ A\tau &= a \\ B\tau^2 &= b \\ C\tau^2 &= c \end{aligned}\right\} \quad [11]$$

Equations [9] and [10] become

$$x^2 + ax + b - \theta^2 + ce^{-x} \cos\theta = 0 \quad [12]$$

$$(2x + a)\theta - ce^{-x} \sin\theta = 0 \quad [13]$$

By plotting these equations in the x, θ plane, it is possible to locate all the intersections of Equations [12] and [13], and thus, for any given values of A, B, C, τ, to determine the behavior of the system. Those solutions involving positive values of x lead to instability; those solutions with negative x represent disturbances that decay. It is the objective of the designer to select values of A, B, and C such that *all* the terms in Equation [8] are damped (or at least the low-frequency terms, since the high frequencies are generally suppressed by viscous dissipation).

A plot of Equations [12] and [13] would look like Fig. 2 for arbitrary values of a, b, and c. Only the positive θ half of the plane need be examined. Both equations are invariant with respect to a change in the sign of θ. Equation [12] yields two branches, a positive one that is continuous and a negative one that exists only in certain ranges of θ; Equation [13] produces a single-valued curve, also intermittent. The encircled intersections in Fig. 2 correspond to the terms in series [8]. The lowest frequency term is obviously the critical one with regard to the over-all stability of the system. An unstable run may exhibit a finite number of low frequency modes, but the modes of higher frequency will be damped.

Certain conditions can be derived for which solutions with positive values of x are impossible. Thus, if $a > c$, the curve given by Equation [13] would lie entirely in the negative x half of the plane. Therefore, this inequality is a sufficient condition for stability. The inequality can be expressed in terms of the appropriate design parameters by means of the defining Equations [7] and [11] and the following relationships:

$$\left.\begin{aligned} \dot{m} &= \rho v_2 A_2 \\ \Delta p &= \frac{1}{2} \rho v_2^2 \left(\frac{A_2^2}{A_3^2} + K\right) \\ c^* &= p_c A_t / \dot{m} \end{aligned}\right\} \quad [14]$$

The condition for stability then becomes

$$\frac{l_2}{p_c} \frac{\dot{m}}{A_2} + \frac{c^* L^*}{RT_c} \frac{2\Delta p}{p_c} > \tau \quad [15]$$

Another useful form of this equation can be obtained by using the theoretical expression for c^*:

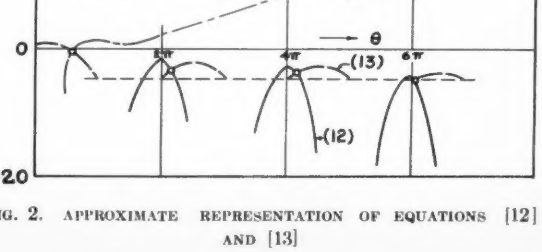

FIG. 2. APPROXIMATE REPRESENTATION OF EQUATIONS [12] AND [13]

[6] The method of solution employed here follows that of N. Minorsky for time lag problems (4).

$$c^* = f(\gamma)\sqrt{RT_c};\ f(\gamma) = \gamma^{-1/2}\left[\frac{\gamma + 1}{2}\right]^{(\gamma + 1)/2(\gamma - 1)} \quad [16]$$

$$f(\gamma) \approx 1.55 \text{ for } 1.1 < \gamma < 1.3$$

By substitution in Equation [15] the inequality takes the form

$$\frac{l_2}{p_c}\frac{\dot{m}}{A_2} + 2.4\,\frac{L^*}{c^*}\frac{2\Delta p}{p_c} > \tau \quad [17]$$

Although this inequality expresses merely a *sufficient condition* for stability, it will appear in the following section, when the magnitudes of the terms are examined for typical cases, that the condition is sufficiently close to be useful as a design criterion.

Another general condition for stability can be derived by combining Equations [12] and [13] and examining the possible roots (θ_n, x_n)

$$\left.\begin{aligned} \theta^4 + \theta^2 f(x) + g(x) &= 0 \\ f(x) &= 2x^2 + 2ax + (a^2 - 2b) \\ g(x) &= (x^2 + ax + b)^2 - c^2e^{-2x} \end{aligned}\right\} \quad [18]$$

The signs of $f(x)$ and $g(x)$ are of interest. The term $(a^2 - 2b)$ can be evaluated in terms of the defining Equations [7] and [11], and it can be seen that it is inherently positive. Consequently, in the positive x domain where our attention is focused, $f(x)$ is positive. Then, θ^2 can be positive only if $g(x)$ is negative, and conversely, it is possible to eliminate real, positive values of θ^2, and hence of θ, by setting $g(x) > 0$ in the positive x domain. This condition can be assured by setting $b > c$. This inequality can be expressed in terms of the design parameters through Equations [7] and [11], yielding the following condition for stability

$$\frac{2\Delta p}{p_c} > 1 \quad [19]$$

As in the case of Equation [15], this inequality represents merely a *sufficient condition* for stability. Clearly, stability is possible even if $2\Delta p/p_c$ is less than unity, by reference to Equation [15].

The frequencies of the oscillations can be evaluated in principle by solving Equation [18] together with either of Equations [12] or [13]. However, consideration of Equation [18] alone can indicate the magnitude of the frequency of the particular mode in series [8] that makes its appearance as the stability conditions [15] and [19] are gradually relaxed. Thus, Equation [18] being quadratic in θ^2, it can be solved explicitly for θ_{cr}, the critically damped frequency

$$\theta_{cr} = \sqrt{-1/2\,f(0) + 1/2\,\sqrt{f(0)^2 - 4g(0)}} \quad [20]$$

If it is supposed that the condition of critical damping occurs when $(2\Delta p/p_c)$ is near to unity, then by setting $(c - b) \ll 1$ and expanding the square root accordingly it can be shown that

$$\omega_{cr} \approx \frac{\left(1 - \dfrac{2\Delta p}{p_c}\right)^{1/2}}{[(l_2\dot{m}/p_cA_2)^2 + (c^*L^*/RT_c)^2]^{1/2}} \quad [21]$$

As expected, $\omega_{cr} \to 0$ as $(2\Delta p/p_c)_{cr} \to 1$.

As a second case, it may be supposed that the condition of critical damping is reached by relaxing condition [15]. Then, c may be replaced by a in Equation [20], and if it is further assumed that $l_2\dot{m}/p_cA_2$ is of the same order as $\dfrac{c^*L^*}{RT_c} \cdot \dfrac{2\Delta p}{p_c}$, it can be shown that:

$$\omega_{cr} \approx \left[\left(\frac{RT_c}{c^*L^*}\right)\left(\frac{p_cA_2}{l_2\dot{m}}\right)\left(\frac{2\Delta p}{p_c} + 1\right)\right]^{1/2} \quad [22]$$

By comparison with condition [15], it is apparent that in this case ω_{cr} is of the order of $2\tau^{-1}$.

A third case of interest may be developed by assuming that $\tau \ll \omega_{cr}^{-1}$. This assumption makes it possible to reconsider the basic differential Equation [6]. Thus, $u(t - \tau)$ can be expanded in a Taylor series:

$$u(t - \tau) = u(t) - \tau u'(t) + \ldots \quad [23]$$

and the differential equation takes a well-known form

$$u'' + (A - C\tau)u' + (B + C)u = 0 \quad [24]$$

The condition for stability is that the coefficient $(A - C\tau)$ shall be positive; this provides the same relation as condition [15], but in this case it is a *necessary condition*, whereas condition [15] was merely a sufficient condition. The frequency in the critically damped situation is simply $\sqrt{B + C}$ which reduces exactly to Equation [22]. (This result requires careful examination in each specific case, however, since $\sqrt{B + C}$ is often of the order of τ^{-1}.)

By inserting typical design values in Equations [20], [21], and [22], it may be concluded that frequencies greater than 200 cycles per sec are unlikely. This supports the opinion expressed in the introduction, that this type of instability (chugging) is a different phenomenon from the high-frequency oscillations (screaming) sometimes observed.

It is of interest to observe that the monopropellant analysis of reference (2) appears as a special case of this analysis. In Equation [6] allow $L^* \to 0$

$$\frac{l_2\dot{m}}{p_cA_2}\,u' + \frac{2\Delta p}{p_c}u + u(t - \tau) = 0 \quad [25]$$

This equation can be identified exactly with Equation [5] of (2). Being of lower order, the differential equation for this limiting case can be solved for the critical condition in simple closed form (in contrast to the system of Equations [12], [13], [20] which do not yield so simple a solution); thus, it is unnecessary to resort to the relatively weaker sufficient conditions. The critical condition for stability then becomes

$$\frac{l_2\dot{m}}{p_cA_2} \cdot \frac{\pi - \cos^{-1}(2\Delta p/p_c)}{\sqrt{1 - (2\Delta p/p_c)^2}} > \tau \quad [26]$$

This is equivalent to Equation [13] of (2). This stability condition is useful whenever $(l_2\dot{m}/p_cA_2)$ is much greater than $(c^*L^*/RT_c)(2\Delta p/p_c)$. It has the advantage of permitting the use of smaller values of $(l_2\dot{m}/p_cA_2)$ than the weaker condition [15].

Discussion of Results

Equations [15] and [19] lead to the following rules for overcoming instability in a liquid propellant rocket system:

(*a*) Increase the pressure difference between supply tank and combustion chamber, either by reducing the area of the injector orifices or by inserting resistance elements in the feed lines.

(*b*) Increase the volume or L^* of the combustion chamber.

(*c*) Increase the length of tubing from supply tank to combustion chamber.

(*d*) Reduce the cross-sectional area of the propellant flow passages leading to the injector (or increase the mass flow per unit area).

(*e*) In a bipropellant system, instability may occur in only one of the two feed systems. Therefore, an important step in eliminating instability is to increase the value of the left side of condition [15] pertaining to the weaker of the two.

(*f*) Reduce the combustion time lag of the propellant. This may require changing over to a more reactive propellant or adding catalytic or combustion-promoting substances. This may be outside the bounds of permissible changes in practice. However, it is possible to change the configuration of the injector (somewhat empirically) or to preheat the propellants to accomplish the same result. One way to accelerate combustion of the propellant is to provide a flow pattern in the combustion chamber in which hot gases are recirculated vigorously to mix with the incoming propellant.

There is experimental evidence that some of these recommendations are in the right direction. It has been reported that an increase in Δp or an increase in L^* has been effective in certain instances in eliminating instability (1). In the author's experience, switching from nitric acid–gasoline to nitric acid–aniline in the identical rocket system removed the instability that had been encountered with the former. (See the remarks in the introduction.) In the case of nitromethane, a redesign of the combustion chamber and relocation of the injectors so as to promote recirculation of hot gases and more prompt vaporization and reaction of the injected liquid has been found to suppress instability.

It is of interest to consider the numerical magnitudes of the terms in the stability condition [15] by inserting typical values of the parameters involved: For example, consider a 5000-lb thrust rocket airplane installation:

Length of tubing from tanks to motor	= 5 ft
Equivalent diameter of tubing	= 2.5 in.
Mass flow of propellant = 25 lb/sec	= 0.78 slug/sec
Chamber pressure	= 400 psia
Tank pressure	= 550 psia
Characteristic velocity (c^*)	= 5000 fps
Gas constant R of chamber gas	= 2000 ft-lb/slug, R
Adiabatic flame temperature	= 5500 R
Characteristic length of motor (L^*)	= 60 in.

$$\tau_{cr} = \frac{5 \times 0.78}{400 \times 4.9} + \frac{5000 \times 5}{2000 \times 5500}\frac{300}{400} = 0.002 + 0.002 = 0.004 \text{ sec}$$

In this case, the system will be stable if the combustion time lag is less than 4 milliseconds. On the other hand, if the same motor is installed in a similar system, but with 30 ft of tubing, the tolerable limit for the time lag is increased to 14 milliseconds. A propellant combination and combustion chamber that chugs in the first installation might be stable in the latter.

It is not unreasonable to expect the actual time lags of practical propellant systems to fall within this range. Although the process of ignition is not quite like that of combustion, the observed ignition lag of self-igniting propellants may be expected to indicate an upper estimate of combustion time lags. Observed ignition lags, under conditions similar to the injection process in a rocket motor, are of the order of 5 to 30 milliseconds, i.e., in the sensitive range (3).

An interesting interpretation of condition [15] can be developed by considering the over-all combustion reaction. The time lag τ may be defined as the interval between the instant of entry of an elementary mass of liquid propellant and the later "instant" when this mass is converted to high-temperature gas, that is, when it exerts its full effect upon a pressure gage connected to the combustion chamber. This time interval τ includes the time required (on the average) for the gasification of the elementary liquid mass and the time required for the gas-phase reaction. The gasification time is not calculable on simple grounds, particularly since the process of vaporization is always accompanied by gas-phase or liquid-phase reactions of unknown character. The gas-phase reaction time, on the other hand, can be estimated (roughly) by equating it to the gas-phase residence time in a combustion chamber of *minimum* volume, i.e., an empirically determined volume such that further reduction results in a serious loss in specific impulse. This residence time is given by:

$$\tau_g \approx \frac{(V_c)_{\min}}{\dot{m}/\rho_{g(\text{mean})}} = \frac{L^*_{\min}c^*}{p_c/\rho_{g(\text{mean})}} = \frac{\rho_{g(\text{mean})}}{\rho_{g(\min)}} \cdot \frac{c^*L^*_{\min}}{RT_c} \quad [27]$$

A reasonable assumption is that $\rho_{g(\text{mean})} \sim \rho_{g(\min)}$; then,

$$\tau_g \approx \frac{c^*L^*_{\min}}{RT_c} \quad [28]$$

Denoting the time in the liquid phase as τ_l, condition [15] takes the following form as a condition for stability:

$$\frac{l_2\dot{m}}{p_cA_2} + \frac{c^*L^*}{RT_c} \cdot \frac{2\Delta p}{p_c} > \tau_l + \frac{c^*L^*_{\min}}{RT_c} \quad [29]$$

The second term on the left side and the corresponding term on the right side are usually of the same order of magnitude, since, in practical designs, $2\Delta p/p_c$ is less than unity and L^* is generally made greater than $L^*_{\min}$. There are sound reasons for such design

practice (based upon considerations other than the control of chugging); therefore, the inequality becomes sensitive to the relative magnitudes of the term on the left describing the momentum of the liquid in the feed lines and the term τ_l on the right.

This comparison indicates that in the case of self-igniting propellants (e.g., nitric acid and hydrazine) chugging is less likely to occur than in the case of non-self-igniting propellants (e.g., nitric acid–kerosene) by virtue of the relatively small gasification time of the former. In fact, chugging may be impossible with certain self-igniting combinations even if l_2 is very small, since the L^* term may insure the inequality. On the other hand, care may be necessary in rockets burning less reactive liquids to provide sufficient momentum in the feed lines.

Condition [15] indicates that, in the trend toward compact liquid rocket installations with short pipe lines, the appearance of instability will become more probable. Of the remedies available to the designer (listed above), those involving changes in the propellant feed system are simpler than those involving changes in the configuration of the rocket motor. The pressure difference Δp may be increased by inserting flow restrictors in the lines, but this has the disadvantage of requiring a higher supply pressure. A more attractive alternative is to increase the length or decrease the area of the feed lines: This would require a smaller increase in supply pressure to achieve the same degree of stability.

In computing the magnitude of the momentum term, if the feed system consists of several passages in series, that term in condition [15] becomes $\frac{\dot{m}}{p_c}\sum_i \frac{l_i}{A_i}$. In such calculations, the momentum of the liquid in the regenerative cooling passages should not be overlooked. In systems having centrifugal pumps instead of pressurized propellant tanks, the theory is directly applicable, but the momentum term should include the suction lines as well as the high-pressure lines. In the case of a system with positive displacement pumps, this theory would deny the possibility of unstable flow (unless a compressible air cavity or elastic tubing is present to permit flow oscillations).

In systems with long feed lines it is possible that the effective length may be somewhat less than the measured length due to compressibility of the liquid, particularly if bubbles are present. A rough calculation indicates that the maximum effective length is of the order of $(\dot{m}/A_2\rho\beta p_c\omega)$, where β is the volume compressibility of the liquid. At 100 cycles per sec, this would be of the order of 10 to 20 ft for the usual steel tubing filled with air-free nitric acid. (However, a more precise analysis including the role of the elasticity of the liquid should be carried out for such cases.)

Caution is required in the application of these stability criteria to cases in which the linearization performed herein may be invalid. In addition to the mathematical linearization above, a physical linearization is tacitly assumed in the statement that τ is a constant independent of pressure.

In conclusion, it is suggested that experiments be conducted to test the correctness of this theory; first, to determine the magnitude of the hypothetical time lag and its dependence on pressure, injector configuration, etc.; and second, to determine whether conditions [15] and [19] are valid conditions for stability.

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Single Flow Jet Engines—A Generalized Treatment

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Generalized performance equations are obtained for single flow air breathing jet engines from entropy considerations. The analysis is simplified by the use of convenient parameters in the description of the modes of compression, combustion, and expansion. The generalized equations are applied to a few cases of current interest, for the purpose of illustration.

AIR-BREATHING jet engines may be classified into two groups: (1) Single-flow machines, in which all the working medium undergoes the same heating process; and (2) multiple-flow machines, in which two or more portions of the working medium undergo partially or wholly different processes.

The purpose of this paper is to present a generalized method of performance analysis of existing or conceivable power plants of the first group. The method is based on the description of the modes of compression, heating, and expansion of the working fluid in terms of parameters that can be varied continuously over their respective ranges of practical significance. The thrust and specific impulse of single-flow jet engines are then derived, in the most general form, as functions of such parameters.

This method yields a direct comparison of the performance of existing single-flow jet engines, together with an indication of the extent to which they may be improved and of what may be expected of other conceivable types of engines of the same group.

Symbols

a = speed of sound, fps
A = cross-sectional area of duct, sq ft
c_p = specific heat at constant pressure, Btu per lb, ° R
c_v = specific heat at constant volume, Btu per lb, ° R
f = frequency of pulsations, cps
g = acceleration due to gravity, ft per sec²
H = heating value of fuel, Btu per lb
I = impulse, lb sec
I_a = air specific impulse (impulse per lb of air), sec
I_c = impulse per cycle, lb sec
$I_f = I_a/\varphi$ = fuel specific impulse (impulse per lb of fuel), sec
J = mechanical equivalent of heat
$\dot{m}$ = mass flow of air through the engine, slugs per sec
M = Mach number
n = exponent of the polytropic describing the heating mode
p = static pressure, lb per sq ft absolute
P = stagnation pressure, lb per sq ft absolute
q = heat added per lb of air, Btu per lb
q_a = total amount of heat added per lb of air, Btu per lb
q_r = total amount of heat rejected per lb of air, Btu per lb
r = compression ratio through compressor
R = gas constant, ft-lb per lb, ° R
s = specific entropy, Btu per lb
t = time, sec
T = thrust, lb
u = velocity, fps
v = specific volume, cu ft per lb
V = volume of heated region, cu ft
γ = ratio of specific heats
$\gamma_c, \gamma_h, \gamma_e$ = specific heats under various temperature conditions
η_c = polytropic compressor efficiency, defined through the equation $r = \left(\frac{\theta_2}{\theta_1}\right)^{\frac{\eta_c\gamma_c}{\gamma_c - 1}}$
η_h = combustion efficiency
η_t = polytropic turbine efficiency, defined through the equation $\frac{P_4}{P_3} = \left(\frac{\theta_4}{\theta_3}\right)^{\frac{\gamma_e}{\eta_t(\gamma_e - 1)}}$
ϑ = static temperature, ° R
θ = stagnation temperature, ° R
λ = product of all downstream-to-upstream stagnation pressure ratios associated with internal drag
ρ = density, slugs per cu ft
σ = product of all downstream-to-upstream stagnation pressure ratios associated with shocks
φ = fuel/air ratio

Subscripts

$_0$ = free stream
$_1$ = downstream end of inlet diffuser; compressor inlet
$_2$ = entrance to heating region
$_3$ = end of heat addition; turbine inlet
$_4$ = turbine outlet
$_e$ = exhaust (at tailpipe exit)

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Assumptions and Approximations

1 M_1 and M_2 are assumed to be low enough that $\theta_1 \approx \vartheta_1$ and $\theta_2 \approx \vartheta_2$.

2 It is stipulated that the amount of heat added and the mechanical work performed by the gas during the heating process be always positive or zero (which means that the specific volume of the fluid will never decrease during the heating process).

3 Heating is assumed to involve no change in the chemical composition of the working fluid.

4 All particles of the working fluid are assumed to undergo the same thermal cycle in each engine considered (except for the purging of residual gases).

5 Changes in the mass rate of flow of the fluid passing through the engine are neglected, and the working fluid is assumed to be air at all sections of the flow path.

6 The effect of temperature on the ratio of specific heats for air is taken into account by the use of mean effective values, as follows:

during compression $\gamma = \gamma_c = 7/5(c_{pc} = 0.24, c_{vc} = 0.171)$
during combustion $\gamma = \gamma_h = 1.36(c_{ph} = 0.259, c_{vh} = 0.19)$
during expansion $\gamma = \gamma_e = 4/3(c_{pe} = 0.275, c_{ve} = 0.206)$

7 Heat losses through the walls of the engine are neglected in all cases.

8 The entire work output of the turbine is assumed to be utilized to drive the compressor.

9 Complete expansion to ambient pressure within the exhaust nozzle is assumed in all cases. In the case of nonsteady-flow engines, where flow conditions change periodically in a nozzle of fixed configuration, this assumption has no meaning as such, except in the subsonic phase of the discharge, and must be interpreted as an approximation. With this assumption, the thrust, defined as the change of momentum of the air which flows through the engine in the unit of time, is $T = \dot{m}(u_e - u_0)$ in the case of steady-flow engines, and $T = \dot{m}(\bar{u}_e - u_0)$, where

$\bar{u}_e = \frac{f}{\dot{m}} \int_0^{\dot{m}/f} u_e dm$, in the case of pulsating-flow engines.

Representation of the Heating Mode

Let the change of state which accompanies heat addition be approximated by the polytropic relation pv^n = constant. This representation is no doubt open to challenge, but so little is known about the actual process of heat addition by combustion that there seems to be little ground for preference in the matter of approximation, so long as the underlying assumptions appear to be tenable in the light of current experience. The advantage of the polytropic approximation is that it provides a parameter—the polytropic exponent n—which may be varied continuously over a wide range and is alone sufficient to describe an infinite variety of heating modes of practical interest.

The First Law of Thermodynamics yields

$$\frac{dq}{d\vartheta} = c_{vh} \frac{n - \gamma_h}{n - 1} \qquad [1]$$

On the other hand, assumption 2 may be expressed in the form

$$dq/d\vartheta \geq c_{vh}$$

which is compatible with Equation [1] only if $n \leq 1.0$. It follows that the range of significance of n in this analysis is from $-\infty$ (isochoric mode) to $+1.0$ (isothermal mode).

From Equation [1] and the polytropic relation $dp/d\vartheta = np/(n - 1)\vartheta$ one obtains

$$\frac{dp}{dq} = \frac{p}{\vartheta} \frac{n}{c_{vh}(n - \gamma_h)} \begin{cases} > 0 \text{ when } n < 0 \text{ or } n > \gamma_h \\ < 0 \text{ when } 0 < n < \gamma_h \end{cases} \qquad [2]$$

Equation [2] shows that, within the range of significance of n in this analysis, the static pressure of the gas decreases as heat is added to it when n is positive and increases when n is negative. Concurrently with this pressure variation, the flow velocity will increase as heat is added if $0 < n < 1.0$ and decrease if $n < 0$.

The extent to which the isochoric mode can be approached without becoming incompatible with the maintenance of stationary flow in the heating region depends primarily on the shape of the heating duct and on the distribution of heat sources within it, but is very limited in any case. Indeed, Equation [1] yields

$$\vartheta_3 = \vartheta_2 + \frac{(n - 1)q_a}{c_{vh}(n - \gamma_h)} \qquad [3]$$

which, within the stipulated range of interest of n, is compatible with the stationary flow condition

$$\theta_2 = \vartheta_2\left(1 + \frac{\gamma_c - 1}{2} M_2^2\right) + \frac{q_a}{c_{ph}} > \vartheta_3$$

only if

$$n > \gamma_h\left[1 - \frac{2(\gamma_h - 1)}{(\gamma_c - 1)} \frac{q_a}{c_{ph}\vartheta_2 M_2^2}\right]^{-1}$$

This inequality expresses a necessary (but not sufficient) condition for the maintenance of stationary flow in the heating region during the heating process. The right-hand member of the inequality is the value of n for which the addition of a dimensionless quantity of heat $q_a/c_{ph}\vartheta_2$ would cause the gas to come to rest from an initial Mach number M_2. It represents, therefore, the limiting value of n for stationary flow. It can be verified that this value is always very close to zero. Therefore, the line of demarcation between stationary and nonstationary processes can, for all practical purposes, be set at $n = 0$ (isobaric mode).

Equation [2] can now be interpreted to mean that stationary flow can be maintained when n is positive and that heating processes in which n is negative will generally result in pressure fluctuations, the amplitude of which will increase with the absolute value of n. It has been shown (1)[1] that, in the nonstationary range, the absolute value of n is—all other conditions being equal—directly proportional to the rate of heat release. The heating mode may, however, be largely affected also by other causes, such as pressure waves or dissipative agents. One may, indeed, also regard as valid the converse interpretation of Equation [2]; namely, that, if the static pressure increases during the heating process, then n is negative. In other words, just as the heating mode may be responsible for the generation of flow fluctuations, so may flow fluctuations which are otherwise excited be responsible for modifications of the heating mode. The parameter n is used here to describe the changes of state which accompany the addition of heat, regardless of whether they are produced by heating alone or also by other causes.

The present analysis will accordingly consider two distinct classes of jet engines, corresponding to two distinct ranges of the parameter n: steady-flow engines which will be analyzed for values of n ranging from $n = 1$ (isothermal heating) to $n = 0$ (isobaric heating); and pulsating-flow engines, the analysis of which will cover all polytropic heating modes from isobaric ($n = 0$) to isochoric ($n = -\infty$).

Exhaust Temperature

From assumption 4, it follows that all the working fluid (with the possible exception of "residual" gases in pulsating-flow engines) undergoes the same total

[1] Numbers in parentheses refer to references on page 126.

specific entropy rise $(s_e - s_0)$ in flowing through the engine. On the other hand, because of assumption 9,

$$s_e - s_0 = \bar{c}_p \ln \frac{\vartheta_e}{\vartheta_0}$$

where

$$\bar{c}_p = \frac{\int_0^e c_p(d\vartheta/\vartheta)}{\int_0^e (d\vartheta/\vartheta)}$$

Assuming, for the sake of simplicity, $\bar{c}_p = c_{ph}$, one obtains

$$\frac{\vartheta_e}{\vartheta_0} = \exp\left(\frac{s_e - s_0}{c_{ph}}\right) \qquad [4]$$

Entropy Rise Through the Engine. Thermal Efficiency

The specific entropy rise $(s_e - s_0)$ will now be calculated as the sum of the following increments:

(a) *Entropy rise through shocks:*

$$\Delta s_\sigma = -\frac{R}{J} \ln \sigma \qquad [5]$$

where σ is the product of all the downstream-to-upstream stagnation pressure ratios across the various shocks which are traversed by a particle of the working fluid as it flows through the engine.

(b) *Entropy rise due to internal drag:* The effect of wall friction and flow separation in subsonic diffusers, sharp changes of cross section, narrow passages and bends, or around such obstacles as fuel injectors, flame holders, etc., is in each case a loss of stagnation pressure without change of stagnation temperature. The entropy rise due to internal drag is therefore

$$\Delta s_\lambda = -\frac{R}{J} \ln \lambda \qquad [6]$$

where λ is the product of all the downstream-to-upstream stagnation pressure ratios associated with such losses within the engine.

(c) *Entropy rise through compressor:*

$$\Delta s_c = c_{pc} \ln \frac{\theta_2}{\theta_1} - \frac{R}{J} \ln r$$

$$= c_{pc}(1 - \eta_c) \ln \frac{\theta_2}{\theta_1} \qquad [7]$$

or

$$\Delta s_c = \frac{R}{J} \cdot \frac{(1 - \eta_c)}{\eta_c} \ln r \qquad [7']$$

(d) *Entropy rise due to heat addition:* Equation [1] yields

$$ds_h = \frac{dq}{\vartheta} = c_{ph} \frac{n - \gamma_h}{\gamma_h(n - 1)} \cdot \frac{d\vartheta}{\vartheta}$$

whence

$$\Delta s_h = c_{ph} \frac{n - \gamma_h}{\gamma_h(n - 1)} \ln \frac{\vartheta_3}{\vartheta_2}$$

Finally, from Equation [3], and with assumption 1, one obtains

$$\Delta s_h = c_{ph} \frac{n - \gamma_h}{\gamma_h(n - 1)} \ln \left[1 + \frac{\gamma_h(n - 1)q_a}{(n - \gamma_h)c_{ph}\theta_2}\right] \qquad [8]$$

It can be verified that, when Δs_h is expressed in this form, $\frac{\partial}{\partial n}(\Delta s_h) > 0$: with a fixed value of $q_a/c_{ph}\theta_2$, Δs_h increases monotonically as n is increased from $-\infty$ to $+1.0$.

In dealing with engines in which material properties impose a limitation on the peak stagnation temperature of the cycle, it is convenient to express Δs_h in terms of θ_3.

In the stationary flow range $(0 < n < 1)$, one has

$$\frac{\theta_2}{\theta_3} = 1 - \frac{q_a}{c_{ph}\theta_3} \qquad [9]$$

From Equations [8] and [9], one obtains

$$\Delta s_h = c_{ph} \frac{n - \gamma_h}{\gamma_h(n - 1)} \ln \left[1 + \frac{\gamma_h(n - 1)}{(n - \gamma_h)\left(1 - \frac{q_a}{c_{ph}\theta_3}\right)}\right] \qquad [8']$$

and it can be verified that, in this range, $\frac{\partial}{\partial n}(\Delta s_h) > 0$.

In the nonstationary flow range $(n < 0)$, Equation [9] no longer applies, but $M_3 < M_2$, and it is therefore permissible, in accordance with assumption 1, to substitute θ_3 for ϑ_3 as well as θ_2 for ϑ_2 in Equation [3]. This equation then becomes a relation between θ_3 and θ_2 which, together with Equation [8], yields

$$\Delta s_h = -c_{ph} \frac{n - \gamma_h}{\gamma_h(n - 1)} \ln \left[1 - \frac{\gamma_h(n - 1)}{(n - \gamma_h)} \frac{q_a}{c_{ph}\theta_3}\right] \qquad [8'']$$

and in this case one finds $\frac{\partial}{\partial n}(\Delta s_h) < 0$. In other words, when $q_a/c_{ph}\theta_3$ is fixed, $\frac{\partial}{\partial n}(\Delta s_h)$ is negative in the nonstationary flow range and positive in the stationary flow range. It follows that the minimum value of the entropy increment Δs_h with the prescribed value of $q_a/c_{ph}\theta_3$ is obtained in the neighborhood of $n = 0$.

(e) *Entropy rise through the turbine:*

$$\Delta s_t = c_{pe} \ln \frac{\theta_4}{\theta_3} - \frac{R}{J} \ln \frac{P_4}{P_3}$$

$$= c_{pe} \frac{1 - \eta_t}{\eta_t} \ln \frac{\theta_3}{\theta_4} \qquad [10]$$

The entropy change due to chemical reaction is neglected in this analysis. The resulting error is of the same order of magnitude as that involved in the assumption of constant specific heats during the combustion process. At any rate, the error depends primarily on the fuel used and will not appreciably affect the relative merits of different types of engines employing the same fuel.

Δs_h is almost always the largest of all entropy increments within the engine, and it is on the magnitude of this increment that the thermal efficiency of the engine primarily depends. In accordance with assumptions 7 and 9, the amount of heat rejected per unit weight of the working fluid in each cycle is $q_r =$

$\bar{c}_p(\vartheta_e - \vartheta_0)$. Therefore, from the definition of thermal efficiency and Equation [4], one obtains

$$\eta_{th} = \frac{q_a - q_r}{q_a}$$
$$= 1 - \frac{\bar{c}_p\vartheta_0}{q_a}\left[\exp\left(\frac{s_e - s_0}{\bar{c}_p}\right) - 1\right] \quad \text{[11]}$$

From inspection of Equation [11] and from the discussion of Equations [8], [8′], and [8″], it may now be concluded that, when all entropy increments except Δs_h are assumed to be constant, the thermal efficiency of a jet engine may be affected by changes of heating mode in two distinctly different manners, depending on whether $q_a/c_{ph}\theta_2$ or $q_a/c_{ph}\theta_3$ is prescribed: in the former case, η_{th} increases monotonically as the heating process approaches the isochoric mode; in the latter case it becomes maximum when the heating mode is approximately isobaric.

Propulsive Efficiency

With the assumption of complete expansion of the working fluid at the exhaust ($p_e = p_0$) the thermal and propulsive efficiencies can be expressed in the forms

$$\eta_{th} = \frac{f}{2q_aJ\dot{m}}\int_0^{\dot{m}/f}(u_e^2 - u_0^2)dm$$

and

$$\eta_p = \frac{2u_0\int_0^{\dot{m}/f}(u_e - u_0)dm}{\int_0^{\dot{m}/f}(u_e^2 - u_0^2)dm}$$

respectively. It is clear that the type of discharge which is most favorable from the standpoint of propulsive efficiency, for a given thermal efficiency and for given u_0, $\dot{m}/f$, and q_a, is that described by the function $u_e = u_e(m)$ which maximizes the momentum flux $\int_0^{\dot{m}/f} u_e dm$ through the exit, with the constraint $\int_0^{\dot{m}/f} u_e^2 dm$ = constant. Euler's equation for the vanishing of the first variation of the integral $\int_0^{\dot{m}/f}(u_e - \Lambda u_e^2)dm$, where Λ is an "arbitrary" constant to be determined from the equation of constraint, yields immediately $u_e = 0.5\Lambda$ = constant as the extremal, indicating that the maximum propulsive efficiency is obtained when all the working fluid leaves the engine at the same velocity. It must be noted that this extremal may be interpreted to represent either a stationary flow at the exhaust or an exhaust velocity which is a "square wave" function of time—the gas being discharged all at the same velocity but intermittently with respect to time.

Stationary Flow Engines

Assuming again, for the sake of simplicity, that the effective mean value of c_p through the engine be $\bar{c}_p = c_{ph}$ (a simplifying assumption which is certainly justified by the degree of approximation of this analysis), the exhaust velocity of a stationary-flow engine can be calculated from the energy relation

$$u_e^2 = u_0^2 + 2gJc_{ph}\left(\frac{q_a}{c_{ph}} + \vartheta_0 - \vartheta_e\right)$$

which, together with Equation [4], yields

$$u_e^2 = u_0^2 + 2gJc_{ph}\left\{\frac{q_a}{c_{ph}} - \vartheta_0\left[\exp\left(\frac{s_e - s_0}{c_{ph}}\right) - 1\right]\right\}$$

and therefore

$$I_a = \frac{T}{\dot{m}g} = \frac{u_e - u_0}{g}$$
$$= \left\{\frac{2Jc_{ph}\vartheta_0}{g}\left[\frac{\gamma_c}{\gamma_h}\frac{(\gamma_h - 1)}{2}M_0^2 + \frac{q_a}{c_{ph}\vartheta_0} - \exp\left(\frac{s_e - s_0}{c_{ph}}\right) + 1\right]\right\}^{1/2} - \frac{u_0}{g}$$

With the values of γ and c_p assumed in this analysis, one obtains

$$I_a = \left\{12.5\vartheta_0\left[1 + 0.185M_0^2 + \frac{q_a}{0.259\vartheta_0} - \exp\left(\frac{s_e - s_0}{c_{ph}}\right)\right]\right\}^{1/2} - \frac{u_0}{32.2} \quad \text{[12]}$$

If heating takes place by combustion, $q_a = H\varphi\eta_{th}$; hence, $I_f = \dfrac{I_aH\eta_{th}}{q_a}$.

Pulsating Flow Engines

The generation of thrust in intermittent jet engines is a periodic or almost periodic process of considerable complexity, because each cycle is affected by the waves which still reverberate through the engine from the previous cycles and similarly affects the cycles which follow. The method of characteristics can be applied to the investigation of the approach to periodic operation, i.e., to such conditions as are finally established when the effect of previous history has become the same for all cycles. A large accumulation of errors is likely to occur, however, in the graphical construction of the extensive characteristic network which is required to arrive at a description of the periodic process. While it may be argued that the accuracy of the results is still within the approximation of the conjectural assumptions which must be made at any rate with regard to the combustion mode, it is clear that with the very use of such approximations the laboriousness of the method becomes unwarranted, regardless of its accuracy. Indeed, there is reason to believe that a comparable degree of over-all accuracy could be obtained by the use of assumptions and approximations which will permit direct treatment of the periodic operation. It is, e.g., more convenient (and perhaps more accurate) to apply the polytropic approximation to what, on the basis of observations, may reasonably be expected to be the combustion process in periodic operation than it is to apply it to the initial explosion only, with an additional set of assumptions as to the effect of pressure waves on the heating mode. It must be noted that the waves which propagate

through the heating region when each explosion takes place and those which are generated in each explosion are, in general, of predominant importance in the economy of ~~the cycle.~~ The simplification ~~which~~ is brought about by the use of the polytropic approximation is particularly important with regard to these waves, because the parameter n describes the changes of state which occur during the combustion process *regardless of their genesis*. Therefore, all waves which propagate through the heating region are implicitly accounted for not only in so far as they modify the heating mode but also as they set out to affect the succeeding phases of the cycle, and previous history need not be given further consideration.

The thrust of intermittent jet engines can then be assumed to be obtained from the periodic discharge of gases from regions where the pressure is periodically raised by explosions, the frequency of the forcing function being approximately the same as the damped natural frequency of the oscillating gas system on which it acts.

With these assumptions, the changes of state or flow parameters at any station of the flow path during one cycle of the periodic operation become identified with the corresponding changes during the first complete fluctuation in the free oscillating discharge which would follow a single explosion of the type considered in the periodic process. In particular, the impulse which is generated in each cycle can be obtained from consideration of this single fluctuation, the initial condition being the same as that which is produced, in the periodic operation, by each explosion.

The analysis by the method of characteristics is thus reduced to computations over a single cycle. The impulse generated in each cycle can then be calculated through time-integration of the momentum transport at the exhaust, for any given free stream velocity:

$$I_c = \int_0^{\dot m/f} (u_e - u_0)dm$$

$$= \int_0^{\tau} \rho_e A_e u_e(u_e - u_0)dt$$

where τ is the time of outflow of the mass $\dot m/f$.

It must be noted that τ does not represent the entire period $1/f$ of the exhaust velocity fluctuation but only that portion of it during which the exhaust velocity is positive. Indeed, if $\dot m/f$ represents all the mass of air which flows into the engine and is brought to rest there in each cycle, then the backflow impulse contribution is already accounted for in the momentum term $-\dfrac{\dot m}{f}u_0$; and if, as is common practice, $\dot m/f$ is taken to represent only the mass of air which is heated in each cycle, then the backflow air is not part of this mass and must be given separate consideration.

The approximations introduced so far simplify the problem to a considerable extent but are not sufficient to make it amenable to analytic solution. In particular, there still remains the important question as to what effect changes of shape of the reservoir and of the discharge duct may have on the impulse which is generated in the discharge of a given mass of gas from given initial reservoir conditions to a given exhaust pressure. What can be said by way of an answer is still tentative. A rather extensive investigation of blow-down discharges, which has recently been carried out at the Cornell Aeronautical Laboratory by the method of characteristics, has revealed that, under the conditions stipulated above, changes of shape have important effects on the duration and detailed history of the blow-down discharge, but hardly any discernible effect on the total impulse which is generated thereby. This observation suggests that the impulse per cycle, with any given reservoir volume, exhaust pressure, and initial reservoir conditions, may—at least in the absence of important entropy gradients in the discharge duct—be substantially independent of the shape of the duct and therefore also of the duration of the blow-down discharge.

Let this conjecture be used as a working hypothesis.

Now, if the discharge time is very long compared to the travel time of waves over the length of the duct, the exhaust velocity can be assumed to be at every instant the same as it would be in the stationary isentropic expansion from the same instantaneous reservoir condition to the same exhaust pressure; in other words, wave phenomena can be neglected and the discharge can be treated as a quasi-stationary process. Therefore, according to our hypothesis, the impulse generated in the blow-down discharge of a gas from a container and through a duct of any shape should be approximately equal to that which would be generated in the quasi-stationary discharge of the same mass of the same gas from the same initial reservoir conditions to the same exhaust pressure.

The operation of intermittent jet engines has been analyzed as a quasi-stationary process by several investigators (2, 3). It is interesting to test the validity of such approximations—and therefore also of our working hypothesis—by comparison of the results of the quasi-stationary discharge analysis with those obtained by the method of characteristics.

As pointed out above, the expansion ends, as far as the generation of impulse is concerned, with the first vanishing of the exhaust velocity. In the analysis of the discharge as a quasi-stationary process, this condition is found to occur when the pressure in the chamber becomes equal to the ambient static pressure p_0, i.e., in the absence of entropy gradients in the discharge flow, when the temperature and density throughout the chamber become equal to the constant exhaust temperature ϑ_e and density ρ_e, respectively. Then, letting subscripts 3 and c denote initial and instantaneous reservoir conditions, respectively, and V the volume occupied by the heated gases when the expansion starts, the impulse per cycle is

$$I_c = \int_{\vartheta_s}^{\vartheta_e} u_e \frac{dm}{d\vartheta_c} d\vartheta_c - V(\rho_3 - \rho_e)u_0$$

the residual mass $V\rho_e$ being scavenged at free-stream velocity,[2] so that no impulse contribution is derived from its discharge. Now,

$$dm = -V \cdot d\rho_c = -\frac{V}{\gamma_e - 1} \frac{\rho_3}{\vartheta_3{}^{1/(\gamma_e - 1)}} \vartheta_c{}^{\frac{2-\gamma_e}{\gamma_e - 1}} d\vartheta_c$$

and

$$u_e{}^2 = \frac{2g\gamma_e R}{\gamma_e - 1}(\theta_c - \vartheta_e) \approx \frac{2g\gamma_e R}{\gamma_e - 1}(\vartheta_c - \vartheta_e)$$

Therefore

$$I_c = \frac{(2g\gamma_e R)^{1/2}}{(\gamma_e - 1)^{3/2}} \cdot \frac{V\rho_3}{\vartheta_3{}^{1/(\gamma_e - 1)}} \int_{\vartheta_e}^{\vartheta_3} \vartheta_c{}^{\frac{2-\gamma_e}{\gamma_e - 1}} (\vartheta_c - \vartheta_e)^{1/2} \cdot d\vartheta_c - V\rho_3\left(1 - \frac{\rho_e}{\rho_3}\right)u_0 \quad [13]$$

With the assumed value 4/3 for γ_e, one obtains

$$I_c = 0.1368 V\rho_3 a \sqrt{\frac{\vartheta_3}{\vartheta_0}}\left[8\left(\frac{\vartheta_e}{\vartheta_3}\right)^2 + 12\frac{\vartheta_e}{\vartheta_3} + 15\right] \times \left(1 - \frac{\vartheta_e}{\vartheta_3}\right)^{3/2} - V\rho_3\left[1 - \left(\frac{\vartheta_e}{\vartheta_3}\right)^3\right] M_0\alpha_0$$

The value of the impulse calculated by means of this equation agrees with the above-mentioned results obtained by the method of characteristics for ducts of various shapes, within the accuracy of these results.

These tests of the validity of our hypothesis are, of course, far from conclusive. While there seems to be little doubt that the quasi-stationary approximation can be used with confidence in the evaluation of certain items of performance, its validity should be supported, if possible, by more rigorous argument.

The air specific impulse is then

$$I_a = \frac{I_c}{V\rho_3 g}$$
$$= \frac{a_0}{g}\left\{0.1368\sqrt{\frac{\vartheta_3}{\vartheta_0}}\left[8\left(\frac{\vartheta_e}{\vartheta_3}\right)^2 + 12\frac{\vartheta_e}{\vartheta_3} + 15\right] \times \left(1 - \frac{\vartheta_e}{\vartheta_3}\right)^{3/2} - \left[1 - \left(\frac{\vartheta_e}{\vartheta_3}\right)^3\right] M_0\right\} \quad [14]$$

again with $q_a = H\varphi\eta_h$ and therefore $I_f = I_a H\eta_h/q_a$, if heating takes place by combustion.

The ratio ϑ_3/ϑ_0 is obtained from Equation [3]:

$$\frac{\vartheta_3}{\vartheta_0} = \frac{\vartheta_3}{\vartheta_2}\frac{\vartheta_2}{\theta_0}\frac{\theta_0}{\vartheta_0} = \left[1 + \frac{(n-1)q_a}{c_{vh}(n - \gamma_h)\vartheta_2}\right]\frac{\vartheta_2}{\theta_0}\frac{\theta_0}{\vartheta_0} \quad [15]$$

and the ratio ϑ_e/ϑ_3 is calculated from Equations [4] to [8], [10], and [15], which, in the case of compressorless pulsating-flow engines ($\Delta s_c = \Delta s_t = 0$), yield

$$\frac{\vartheta_e}{\vartheta_3} = \frac{\vartheta_e}{\vartheta_0}\frac{\vartheta_0}{\theta_0}\frac{\theta_0}{\vartheta_2}\frac{\vartheta_2}{\vartheta_3}$$
$$= (\sigma\lambda)^{-\frac{\gamma_h - 1}{\gamma_h}}\left[1 + \frac{(n-1)q_a}{c_{vh}(n - \gamma_h)\vartheta_2}\right]^{\frac{n(\gamma_h - 1)}{\gamma_h(1-n)}}\frac{\vartheta_0}{\theta_0}\frac{\theta_0}{\vartheta_2} \quad [16]$$

The beneficial effect of an increase of the ratio ϑ_2/θ_0, i.e., of precompression of the charge beyond the ram level, can be evaluated by means of these equations. It must be noted that it is only in nonstationary flow that such precompression can be produced without the aid of a mechanical compressor. This possibility must indeed be regarded as one of the salient features of pulsating-flow jet engines.

The "hammer effect," resulting from rapid interruption of a flow, illustrates this possibility. Let the initial conditions of a flow in a straight tube be $u = u_i$ and $a = a_i$. Interruption of the flow by rapid closure of a gate generates an upstream propagating pressure wave (hammer wave). Let subscript H refer to the flow region between this wave and the gate. Then, $u_H = 0$. The compression ratio produced in this manner, assuming isentropic changes of state, can be computed from the relation (4)

$$a_H - a_i = \frac{\gamma - 1}{2}(u_i - u_H) = \frac{\gamma - 1}{2}u_i$$

whence

$$\frac{p_H}{p_i} = \left(\frac{a_H}{a_i}\right)^{\frac{2\gamma}{\gamma - 1}} = \left(1 + \frac{\gamma - 1}{2}M_i\right)^{\frac{2\gamma}{\gamma - 1}}$$

It can be verified that $p_H > P_i$ if $M_i < \dfrac{4}{3 - \gamma}$. The hammer compression effect is of course lower where the hammer wave has steepened to form a shock.

Another way of producing precompression in pulsating-flow engines is by utilization of the waves which are generated in the combustion process. Equation [2] reveals that a nonstationary polytropic heating process is always accompanied by a pressure rise; and that this rise increases monotonically as the heating process approaches the isochoric mode. Therefore, intermittent combustion is a source of pressure waves which can be used to precompress the fresh charge.

The manner in which pressure waves may be timed and utilized to precompress a charge depends in the first place on the mode of combustion. In the preceding paragraphs, combustion was assumed to take place uniformly throughout the burning region—a condition which is perhaps approached in actual operation when ignition takes place more or less simultaneously at several points in the combustion chamber, or when the flame front is broken up by turbulence into a number of scattered burning "islands." In this case, the pressure is found to rise uniformly in the burning gas, generating pressure waves which are propagated and reflected in the surrounding regions and may be utilized to precompress the fresh charge in the following cycle.

On the other hand, any departure from the condition of uniformity assumed in the preceding paragraphs will introduce a certain amount of precompression *within the cycle*, because the pressure waves generated by the burning portions of the mixture will precompress

[2] **Failure to account for the cost of scavenging led to erroneous results in reference (1). The author is indebted to R. Weatherston for calling his attention to this omission.**

the portions that are yet to be burned. An extreme case of wave precompression of the latter type is that which occurs in the presence of a plane normal flame front propagating axially through a duct filled with a combustible mixture. In this case, and assuming that the change of state across the flame be isobaric, the wave precompression ratio is found to be (1)

$$\frac{p_2}{p_1} = \left(1 + \frac{\gamma_c - 1}{2} U \frac{q_a}{c_{ph}\vartheta_1}\right)^{\frac{\gamma_c}{\gamma_c - 1}} \qquad [17]$$

where U is the ratio between the rate of flame propagation relative to the unburned gas and the velocity of sound in the unburned gas. Utilization of this mode of precompression was recently attempted by Kahane in the "intermittent ramjet" (5).

Equations [14] to [16] can be used to calculate the air and fuel specific impulse of a large class of single flow intermittent-jet engines. There exists, however, a class of engines in which the thrust-generating discharge departs considerably from the analytical model used in the derivation of these equations. This special class is that of the engines in which nonstationary processes are confined to those regions in which they can be utilized to increase the thermal efficiency, while substantially stationary-flow conditions are maintained at the intake and exhaust, to reduce the intake shock losses and to obtain maximum propulsive efficiency (see page 118 under "Propulsive Efficiency"). The performance of any engine of this type must again be calculated by Equation [12], which applies to any engine in which both inflow and outflow are stationary, regardless of the character of the flow within the engine.

Examples

The generalized theory is applied in this section for the purpose of illustration to the following families of power plants: (*a*) ramjets and (*b*) turbojets, with any stationary-flow heating mode; (*c*) pulsejets, with any nonstationary-flow heating mode; and (*d*) wave engines, with isobaric combustion.

The results of the computations are summarized by charts for each of the engines considered, with the fuel specific impulse as the ordinate and the air specific impulse as the abscissa. Lines of constant air/fuel ratio ($1/\varphi$) are also shown in the charts. These are straight lines passing through the origin, because $I_f = I_a/\varphi$.

The following sets of numerical values, in addition to those listed under assumption 6, were used in all performance calculations: $\lambda = 0.9$, $\eta_c = 0.85$, $\eta_t = 0.80$, $\eta_h = 0.90$, $H = 19{,}000$ Btu/lb, and $\vartheta_0 = 416°$ R (corresponding to $a_0 = 1000$ fps and to an altitude of about 29,000 ft in NACA standard atmosphere).

The transition from supersonic to subsonic flow in the inlet diffuser was assumed to occur through a single normal shock, and in all cases the diffuser configuration

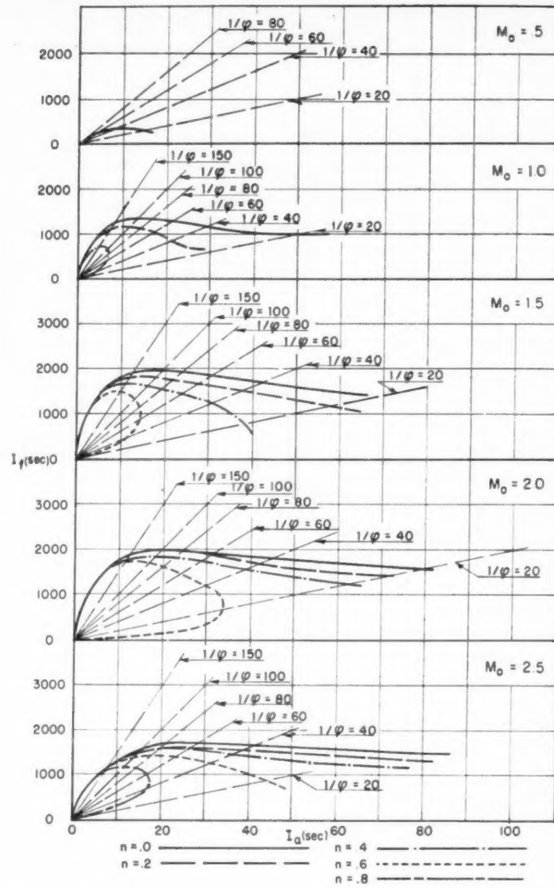

FIG. 1. FUEL SPECIFIC IMPULSE AND AIR SPECIFIC IMPULSE OF RAMJET

was assumed to be that described in (6). In stationary-flow engines the shock was assumed to be located at the throat of the diffuser. In nonstationary-flow engines the moving shock was assumed to be retained within the diverging portion of the diffuser, and its mean effective strength was assumed to be that of a normal shock occurring at the free-stream Mach number.

The Ramjet: The ramjet can be broadly defined as any stationary flow ($n \geq 0$) compressorless ($\Delta s_c = \Delta s_t = 0$; $\theta_2 = \theta_0$) air-breathing jet engine. Accordingly,

$$\exp\left(\frac{s_e - s_0}{c_{ph}}\right) = (\sigma\lambda)^{-\frac{\gamma_h - 1}{\gamma_h}} \left[1 + \frac{(n-1)q_a}{c_{vh}(n - \gamma_h)\theta_0}\right]^{\frac{n - \gamma_h}{\gamma_h(n-1)}} \qquad [18]$$

The performance of the ramjet can be calculated by means of Equations [12] and [18] for any set of values of n, q_a, ϑ_0, and M_0. The results of such calculations are presented in Fig. 1. The performance of the ramjet is seen to depend to a considerable extent on the value of n, all other parameters being equal.

It is interesting to determine where the ramjet with the commonly used constant-area burner would fit

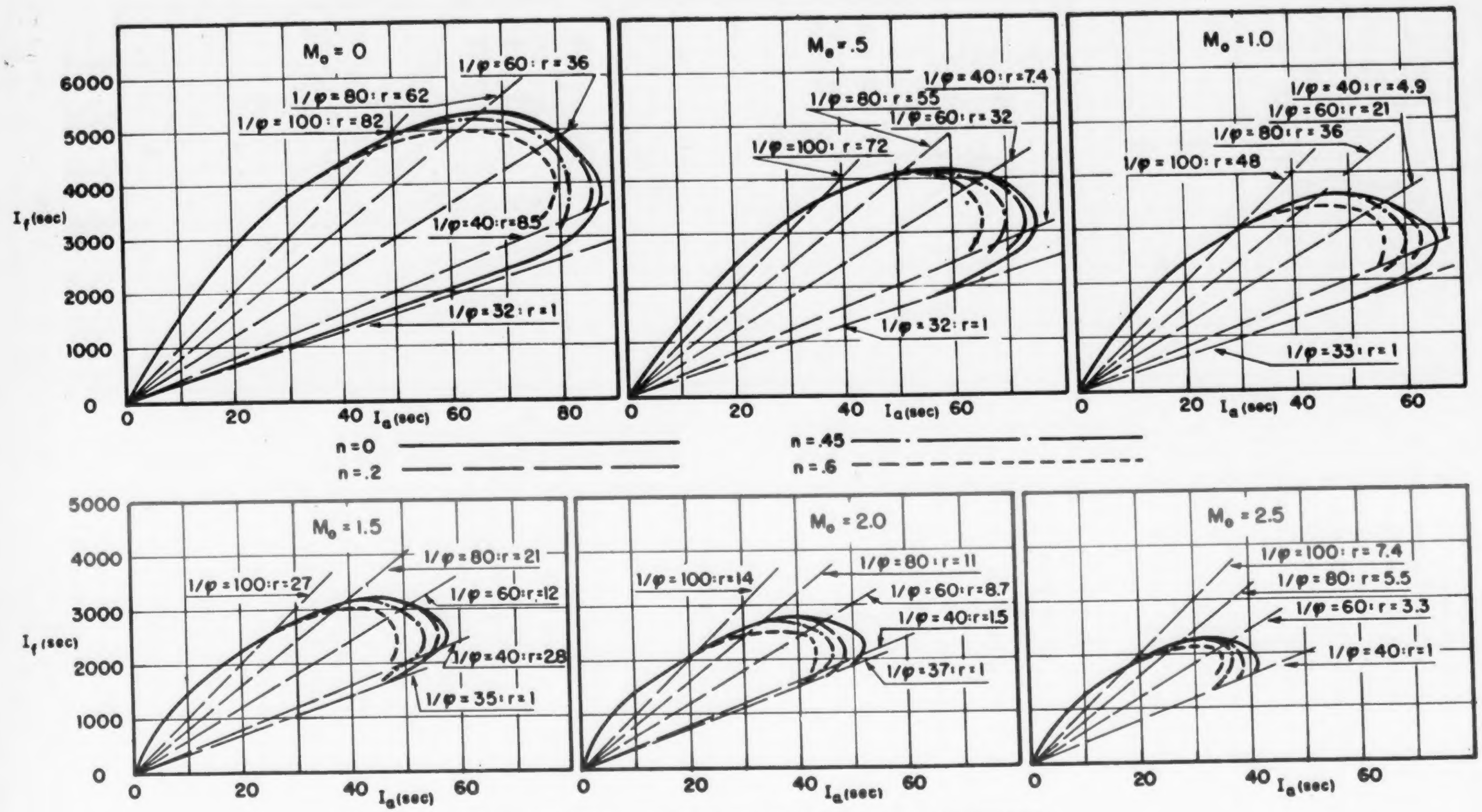

FIG. 2 FUEL SPECIFIC IMPULSE AND AIR SPECIFIC IMPULSE OF TURBOJET

in this chart. Stationary heating of a flow in a constant-area duct is not a polytropic process, but for the infinitesimal change of state at any point the function $p(\rho)$ may be replaced by a polytropic tangent, in the form $dp/d\rho = np/\rho$, where n will of course vary continuously along the heating region. Now, in stationary flow in a constant-area duct one has $dp/\rho = -u \cdot du$ and $du/u = -d\rho/\rho$, hence $dp/d\rho = u^2$. Therefore, $u^2 = np/\rho = na^2/\gamma$, whence $n = \gamma M^2$. The Mach number in the combustion chamber increases from a very low upstream value M_2 to a downstream value M_3 which may be as high as 1.0. Concurrently, n will increase through the heating region from an upstream value very close to zero, through the isothermal value 1.0 at $M = 1/\sqrt{\gamma}$, to the isentropic value γ at $M = 1.0$. It can be verified, however (7), that most of the entropy rise takes place in the low-Mach-number low-temperature region of the flow. Therefore, as far as the total entropy rise is concerned, a stationary heating process in a constant-area duct with a low inlet Mach number is equivalent to a polytropic process which departs only slightly from the isobaric ($n = 0$).

Fig. 1 shows that the optimum heating mode is the isobaric at all flight Mach numbers. It is evident, then, that little room is left for improvement of ramjet performance through modifications of the heating mode. In fact, a more nearly isobaric process would require a diverging combustion chamber, and the consequent increase of external drag could easily offset the slight thrust advantage. On the other hand, some improvement of ramjet performance can be obtained by reducing shock losses and internal drag and by increasing the combustion efficiency. Considering the possibility of such improvements, it seems reasonable to regard the curves for $n = 0$ as fairly representative of the potential performance of ramjets.

In general, the trend of the curves of potential ramjet performance is similar to that of the performance of conventional ramjets as calculated by the usual methods of analysis. With extremely lean mixtures, the fixed losses (shock and friction) are predominant, therefore the fuel specific impulse increases as the energy input is increased. With richer mixtures, the entropy rise due to heat addition is predominant, and therefore an increase of heat input causes the fuel specific impulse to decrease, although the air specific impulse continues to increase. The maximum of I_f occurs at very low values of I_a, but the latter may be increased considerably without too much sacrifice in fuel economy. The highest values of I_f are obtained at flight Mach numbers of about 2, with the inlet diffuser considered in this analysis. The use of improved diffusers would have little effect on the performance at lower Mach numbers, but would increase somewhat the flight Mach number for maximum fuel economy.

The Turbojet: In the turbojet, $n \geq 0$ and $\theta_1 = \theta_0$ as in the ramjet, but $\theta_2 > \theta_1$. Assumption 8 establishes the condition

$$c_{pe}(\theta_3 - \theta_4) = c_{pc}(\theta_2 - \theta_0)$$

Therefore,

$$\exp\left(\frac{s_c - s_0}{c_{ph}}\right) = (\sigma\lambda)^{\frac{1-\gamma_h}{\gamma_h}} \left(\frac{\theta_3 - (q_a/c_{ph})}{\theta_0}\right)^{\frac{c_{pc}}{c_{ph}}(1-\eta_c)} \cdot \frac{\left[1 + \dfrac{(n-1)q_a}{c_{vh}(n-\gamma_h)[\theta_3 - (q_a/c_{ph})]}\right]^{\frac{n-\gamma_h}{\gamma_h(n-1)}}}{\left[1 - \dfrac{c_{pc}}{c_{pe}} \dfrac{[\theta_3 - (q_a/c_{ph}) - \theta_0]}{\theta_3}\right]^{\frac{c_{pe}}{c_{ph}} \frac{(1-\eta_t)}{\eta_t}}} \quad \ldots [19]$$

The performance of the turbojet can be calculated by means of Equations [12] and [19]. Fig. 2 is a plot of the results of such calculations, in which the stagnation temperature at the turbine inlet (θ_3) was assumed to be 2500 R. This value is high in comparison with current practice but may well be attained and even exceeded in the near future, as a result of improved blade-cooling techniques. Then

$$\theta_2 = \theta_3 - \frac{q_a}{c_{ph}} = 2500 - \frac{19{,}000 \times 0.90 \times \varphi}{0.259} = 2500 - 66{,}000\,\frac{I_a}{I_f}$$

and

$$r = \left(\frac{\theta_2}{\theta_0}\right)^{\frac{\eta_c\gamma_c}{\gamma_c - 1}} = \left[\frac{1}{\theta_0}\left(2500 - 66{,}000\,\frac{I_a}{I_f}\right)\right]^{2.975}$$

Therefore, in the (I_a, I_f) plane for any given flight Mach number, the loci of points corresponding to the same compression ratio are straight lines passing through the origin. The values of r are indicated in Fig. 2 alongside those of the air/fuel ratio on the constant $-\varphi$ lines.

The lowest significant value of r is of course 1.0, since, when $r = 1.0$, the turbojet becomes a ramjet. On the high side, the establishment of any limitation on the value of r would be outside the scope of the present analysis. Therefore, no such limitation has been set in this calculation, and it will be noted that, as a result of this omission, the performance curves have been extended to regions where the specified conditions would impose compression ratios far above the values which are currently used.

The best performance is obtained at all flight Mach numbers with $n = 0$, but the effect of small departures from the isobaric heating mode is relatively unimportant and vanishes completely when the air/fuel ratio and the compression ratio are very high. Therefore, as in the case of the ramjet, little if any performance improvement could be derived in the turbojet from modifications of the heating mode. Also, the air/fuel ratio which is currently used in turbojets (about 60) appears to be a good compromise between those required for maximum fuel economy (70 to 80) and for maximum specific thrust (about 40). The performance superiority of the model considered in the calculations over present-day turbojets can therefore

be attributed almost entirely to the higher permissible temperature at the turbine inlet. The higher temperature is established through the use of higher compression ratios, without substantial change in mixture composition. The use of richer mixtures is advisable only when a high specific thrust is desired, even at a sacrifice of fuel economy, as in take-off or during acceleration in flight. Thus, e.g., the same compression ratio (about 9) would be required for maximum specific thrust at $M_0 = 0$ (take-off) and for minimum fuel consumption at $M_0 \approx 2$; but the air/fuel ratio would have to be increased, during the acceleration, from about 40 at take-off to about 60 at the cruising Mach number.

The Pulsejet: The pulsejet has been defined (8) as a compressorless pulsating-flow engine which "derives its thrust from a natural oscillation of a valved aeroduct." The valveless pulsejet may accrodingly be described as deriving its thrust from a natural oscillation of a valveless aeroduct. The principle of operation is essentially the same for both types of power plant and their distinguishing feature, in the compressorless pulsating-flow class, in a complete or almost complete absence of wave precompression. These power plants will, therefore, be characterized in the present analysis by the condition $\vartheta_2 = \vartheta_1 = \theta_0$, which must be introduced in Equations [14] to [16] for the computation of their performance.

It must be noted that Δs_σ is not only the entropy rise through the inlet diffuser shock, but also that associated with the pressure waves which are generated by each explosion. Only shock waves are significant in this respect, and it should be borne in mind that this entropy increment must be considered only in so far as it results from wave phenomena occurring outside of the burning region, because the effect of pressure waves within this region is already accounted for in the description of the heating mode. Therefore, consideration must be given to the additional entropy term only if the heating mode is such as to produce pressure waves that are steep enough to form early shocks and, thus, to affect a significant portion of the nonburning region.

In general, the entropy rise caused by these shocks is found to be insignificant unless the rate of heat release is very high, i.e., unless the absolute value of n is very large. In the limiting case of constant-volume combustion, $n = -\infty$, it has been shown (1) that this entropy increment is approximately equivalent to that occurring across a single normal shock propagating at a Mach number $M = \vartheta_3/\vartheta_e$.

The calculated performance of the pulsejet is summarized in Fig. 3. The effect of changes of heating mode is pronounced only in the lower range of $|n|$.

Since it has been shown (1) that $|n|$ is almost linearly proportional to the rate of heat release, it follows that it should become decreasingly profitable to seek performance improvements by increasing the rate of heat release. As a matter of fact, only at the lower Mach numbers is the most economical heating process very nearly isochoric: as the flight Mach number increases, the importance of the shock losses associated with high heating rates also increases and therefore the optimum $|n|$ decreases.

The chart for $M_0 = 0$ is of limited significance because in static operation and at low flight speeds a considerable amount of cold air is drawn into the tailpipe of the pulsejet in the backflow which follows the blow-down discharge. The propulsive efficiency is thereby increased because of the increased density of the tailpipe gases and of the consequent redistribution of the outflowing kinetic energy over a larger mass of gas.

A gross evaluation of this increase of propulsive efficiency, which is called "internal augmentation," can be made as follows. Let average values be indicated by bars. Then, if the energy transfer could be effected without losses, $\bar{u}_e^2$ would be inversely proportional to ρ_e and therefore $\rho_e\bar{u}_e$ would be approximately directly proportional to $\sqrt{\rho_e}$. The magnitude of this effect is, therefore, very appreciable. As the flight speed increases the augmentation effect decreases rapidly, not only because backflow is reduced but also because the momentum lost by the augmentation air as it is brought to rest into the tailpipe from the surrounding stream is in any case proportional to the velocity of this stream.

The chart for $M_0 = 0$ in Fig. 3 must therefore be regarded as quite pessimistic. Indeed, the measured fuel specific impulse of some pulsejets in static operation (9) has already exceeded the best theoretical values indicated by this chart.

Assuming that the rate of heat release be in each case that corresponding to the optimum value of n, comparison of Figs. 1 and 3 reveals the interesting fact that the potential superiority of the pulsejet over the ramjet is not limited to subsonic flight conditions but extends well into the supersonic domain for most practical values of I_a. At the highest flight Mach numbers considered in this calculation, the performance of the pulsejet is still comparable, if not superior to that of the ramjet when the required air specific impulse is very large. It is also interesting to note (Fig. 3) that the rate of heat release required for optimum pulsejet performance at the highest Mach number is well within the range of possible improvement. This analysis is, of course, based on the assumption that the great difficulties which are now encountered in the high-speed operation of pulsejets can be surmounted. These difficulties are largely due to the inability of pulsejets to sustain ram pressure in the combustion chamber during the charging phase of the cycle. There is reason to believe, however, that this situation can be remedied, either by the introduction of a constriction in the tailpipe (to "choke" the outflow over a

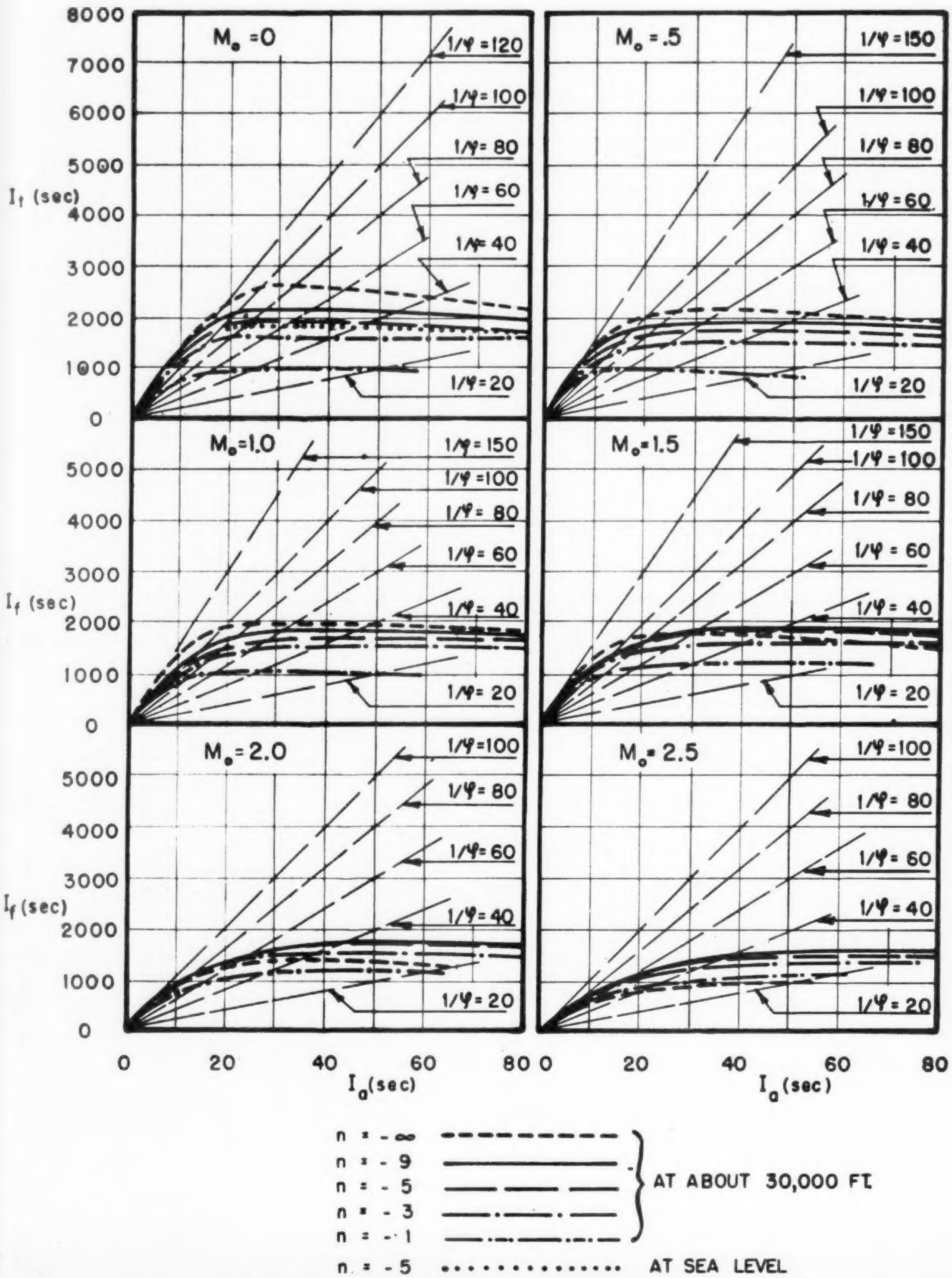

FIG. 3 FUEL SPECIFIC IMPULSE AND AIR SPECIFIC IMPULSE OF PULSE JET

longer portion of the cycle) or by shrouding the tail end to produce a local rise of static pressure; and perhaps also by other means.

In Fig. 3 the static performance curve for $n = -5$ at sea level is also plotted in the chart for $M_0 = 0$, to illustrate the effect of altitude on specific impulse. This effect is seen to be quite small.

Wave Engines: This designation is now customarily applied only to compressorless jet engines in which the fresh charge is precompressed by wave processes which are generated and/or controlled by timed valving (10).

The entropy increment Δs_σ rates particular attention in this case because the process of wave precompression may involve shock waves of considerable strength. The losses associated with these shocks represent the cost of energy transfer from the hot gases to the fresh charge, the wave-engine counterpart of the turbine and compressor losses in turbojets. It must be remembered that a train of weak pressure waves can be far more efficient as a compressor than a single strong shock producing the same pressure rise. Therefore, important improvements of precompression efficiency may be expected to result from refined techniques of shock-formation control.

Now let the following conditions be assumed: (*a*) Wave precompression takes place through ν shocks of equal strength; and (*b*) heat is added at constant pressure following wave precompression. Let δ_i and σ_i denote the downstream-to-upstream static temperature ratio and stagnation pressure ratio, respectively, through each of the ν shocks. Then

$$\delta_i^{\nu} = \frac{\vartheta_2}{\vartheta_1}$$

so that

$$\Delta s_\sigma = -\frac{R}{J}\ln(\sigma_0\sigma_i^{\nu})$$

$$= -\frac{R}{J}\ln\left[\sigma_0 \cdot \sigma_i^{\frac{\ln(\vartheta_2/\vartheta_1)}{\ln \delta_i}}\right]$$

$$= -\frac{R}{J}\ln\left[\sigma_0 \cdot \left(\frac{\vartheta_2}{\vartheta_1}\right)^{\frac{\ln \sigma_i}{\ln \delta_i}}\right]$$

where σ_0 is the stagnation pressure ratio across the stationary shock in the inlet diffuser; and $\vartheta_2 = \vartheta_3 - \frac{q_a}{c_{ph}}$.

Within the approximation of this analysis, $\vartheta_1 = \theta_0$, $\vartheta_2 = \theta_2$, and $\vartheta_3 = \theta_3$. Therefore,

$$\exp\left(\frac{s_e - s_0}{c_{ph}}\right) = (\sigma_0\lambda)^{-\frac{\gamma_h - 1}{\gamma_h}}\left(\frac{\theta_3}{\theta_0}\right)^{-\frac{\gamma_h - 1}{\gamma_h}\frac{\ln \sigma_i}{\ln \delta_i}} \cdot \left(1 - \frac{q_a}{c_{ph}\theta_3}\right)^{-\frac{\gamma_h - 1}{\gamma_h}\cdot\frac{\ln \sigma_i}{\ln \delta_i} - 1} \quad [20]$$

Usually, the exhaust velocity from each element of a wave engine is substantially a square-wave function of time, and the engine is made up of a multiplicity of elements operating out of phase, so that the exhaust from the engine is substantially stationary. Similarly, the air intake in the various elements takes place in such close succession from the common inlet diffuser that the flow at the engine intake may be regarded as stationary. Therefore the performance equation for the wave engine is Equation [12], in which the entropy term is now obtained from Equation [20]. The calculated performance, for $\theta_3 = 4500°$ R and $\delta_i = 1.5$ (which corresponds (11) to $\sigma_i = 0.83$), is plotted in Fig. 4. The potential performance of the wave engine is seen to be very promising, at least from the standpoint of fuel economy.

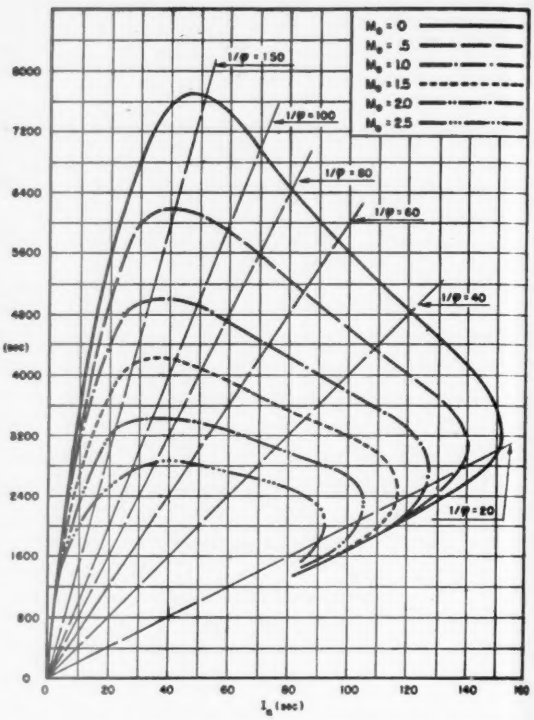

FIG. 4 FUEL SPECIFIC IMPULSE AND AIR SPECIFIC IMPULSE OF WAVE ENGINE

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(*Continued on page 131*)

Approximate Calculations of Specific Heats for Polyatomic Gases

By R. V. MEGHREBLIAN

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This paper presents a method for the approximate calculation of the ideal specific heats of polyatomic molecules occurring in rocket combustion processes. The method is based on the assignment of characteristic frequencies to individual interatomic bonds, the simplification consisting in the use of the same characteristic frequency for a particular bond irrespective of the over-all structure of the molecule in which it occurs. The method can be applied to molecules for which precise spectroscopic data are not available.

Introduction

THE thermodynamic functions of gases can be calculated from spectroscopic data by using standard statistical methods (1, 2).[1] At moderate temperatures the existence of excited electronic states for polyatomic molecules may be ignored. Furthermore, interactions between vibration and rotation make only small contributions to the total energy. Finally, the rotational motion of polyatomic gaseous molecules may be assumed to be fully excited for temperatures of interest in connection with calculations on rocket engines.

It is well known that the specific heat at constant pressure C_p of polyatomic gases, subject to the approximations listed above, is given by either of the two relations

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[1] Numbers in parentheses refer to references at end of paper.

$$C_p = \frac{7}{2}R + R\sum_{j=1}^{3n-5} E(u_j) \quad \text{....(linear molecules)....} \ [1]$$

or

$$C_p = 4R + R\sum_{j=1}^{3n-6} E(u_j) \quad \text{....(nonlinear molecules)....} \ [2]$$

Here R represents the molar gas constant, n is the number of atoms in the molecule under discussion, and $E(u_j)$ represents the Einstein function (3) for the jth vibration frequency. The characteristic numbers u_j are determined from the normal vibration frequencies ν_j according to the relation

$$u_j = h\nu_j/kT$$

where h and k represent Planck's and Boltzmann's constants, respectively, and T is the absolute temperature.

Characteristic Vibration Frequencies and the Calculation of Heat Capacities

It is possible to obtain approximate values for the specific heats by utilizing the fact that characteristic vibration frequencies between two atoms are not usually strongly dependent on the atomic groups to which the vibrating atoms are attached (4). Thus, one can associate with a given chemical bond two basically different types of motion, viz., stretching and bending

TABLE 1. CHARACTERISTIC WAVE NUMBERS OF CHEMICAL BONDS

Bond	Stretching vibration (cm^{-1})	Bending vibration (cm^{-1})	Bond	Stretching vibration (cm^{-1})	Bending vibration (cm^{-1})
C—C (aliphatic)	910	650	H—O	3500	1350
C—C (aromatic)	1500	600	H—S	2570	860
C═C	1200	910	S═O	1250	520
C≡C	2080	375	S—C	690	280
C—H	3000	1050	N—N bond in N_2O	2220	590
			N—N bond in N_2O_4	280	500
C—O	1030	1120	N—N	1000	900
C═O	1740	780	N—H	3300	1200
C—N	1000	450	N—O	1270	660
C≡N	2220	240	N═O	1470	650
C—Cl	650	260	Si—H	2180	930
C—F	1050	1200	Si—O	1050	400
C—Br	610	950	B—H	2550	1150
C—I	530	880	B—F	890	690
			B—N[a]		

[a] The information available on compounds with B—N bonds was inadequate for assigning average normal vibration frequencies. However, calculations of C_p for borazole ($B_3N_3H_6$), based on the assignments 1350 cm^{-1} and 500 cm^{-1}, have yielded reasonable agreements with the more precise C_p data obtained by Crawford and Edsall (4).

vibrations. This correlation is useful (5) in spite of the fact that the normal vibration frequencies represent motions of the entire molecular structure rather than of two isolated atoms.

Previous investigators (5) have compiled a list of characteristic frequencies for some of the important chemical bonds. However, these lists are inadequate for performing extensive calculations on propellants. Consequently an investigation was undertaken to extend this information to additional bonds, and to revise the frequency assignments by using the best available spectroscopic data. A large number of different compounds (4) were examined in order to obtain reasonable values for the characteristic frequencies.

The suggested assignments, in wave numbers, are listed in Table 1. The small and large values correspond, respectively, to the bending and stretching vibrations. The numerical values given in Table 1 do not differ greatly from the assignments of other investigators for the chemical bonds for which previous assignments are available (5).

Specific heats of polyatomic molecules may be calculated from Equations [1] or [2] by using the data of Table 1 together with an averaging procedure introduced by Bennewitz and Rossner (5), which has not been applied extensively by other investigators. It is found, in general, that the sum of the stretching and bending frequencies will be fewer than the $3n - 6$, or $3n$-5, normal vibration frequencies. This discrepancy is, of course, the result of the existence of complex group vibrations in the molecule. The suggested procedure for evaluating C_p is to account for the remaining unknown frequencies by assigning to the specific heat contribution of the undetermined frequencies an average value based on the contribution of the known bending vibrations.[2] Thus, the necessary adjustment is made by applying a correction factor (the ratio of the total number of nonstretching vibrations to the number of bending vibrations) to the C_p contribution of the bending vibrations. Illustrative examples, showing the method of calculation, may be found in the paper of Bennewitz and Rossner (5).

The results of sample calculations of specific heats are summarized in Table 2. Both theoretical (based on actual frequency assignments) as well as experimental values of C_p have been included. It is expected that results comparable to those indicated in Table 2 will be obtained for most polyatomic gases. For practical purposes, the calculated results should have acceptable accuracy. The principal advantage of the present method of calculation is the relative ease with which one can evaluate C_p for polyatomic gases.

TABLE 2. SAMPLE CALCULATIONS OF SPECIFIC HEATS OF GASEOUS COMPOUNDS

Molecule	Temperature (C)	C_p (theoretical)	C_p (approximate)	C_p (experimental)
CH_3OH	137	...	12.9	11.8
N_2O_4	27	15.86	15.63	...
NH_3	0	...	8.4	8.7[a]
	300	...	11.06	10.27
	500	...	12.51	11.31
	700	...	13.74	12.92
	850	...	14.52	13.22
	1000	...	15.15	14.65
C_6H_6	20	19.1	20	21.8[b]
	100	24.5	25.7	25.8
	350	38.35	39.45	38.9
CH_3NH_2	10	11.72	11.85	11.72[c]
	25	...	12.29	12.41
	50	12.62	13.04	13.79
C_2H_6	64.5	13.6	14.1	13.4[d]
N_2O	20	10.24	10.14	9.24[e]
$(CH_3)_2NH$	10	...	15.8	15.8[f]
	50	...	18	18.66
$CHCl_2Br$	27	16.2	16.19	...
	427	21.48	21.72	...
C_2H_3CN	25	15.24	15.04	...
	327	23.11	23.7	..

[a] Cf. Ref. 6, p. 1228.
[b] Cf. Ref. 7, p. 1275.
[c] Cf. Ref. 7, p. 2317.
[d] Cf. Ref. 7, p. 2316.
[e] Cf. Ref. 8, p. 1215.
[f] Cf. Ref. 7, p. 2317.

Acknowledgment

The author takes pleasure in expressing his appreciation for the assistance and helpful criticism of Dr. S. S. Penner and Dr. H. S. Tsien.

[2] The reason for basing the correction on the known bending vibrations, rather than on an average of all the vibrations, is that experience has shown that the unevaluated frequencies are generally low, comparable to the bending vibrations.

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(Continued on page 131)

The Hypergolic Reaction of Dicyclopentadiene with White Fuming Nitric Acid[1]

By C. H. TRENT[2] and M. J. ZUCROW[3]

The photographic study of the hypergolic reaction between dicyclopentadiene and anhydrous nitric acid indicates that a solid phase is formed prior to ignition and that the ignition of the propellant is propagated from the solid phase. The latter has been isolated. Although the identity of the solid phase remains unknown, characterization tests have indicated that it is a nitrated polymer containing the nitro group and the nitrate ester group.

THE discovery that dicyclopentadiene was hypergolic with nitric acid led to the further study of that hydrocarbon for the purpose of obtaining information on the behavior of hypergolic hydrocarbon compounds.

Photographic Study

When liquid dicyclopentadiene and white fuming nitric acid were reacted in an inert atmosphere of dry nitrogen, spontaneous ignition occurred in spite of the absence of atmospheric oxygen. Furthermore, dicyclopentadiene reacts hypergolically even when the nitric acid is added to it dropwise (approximately 10 drops per sec) in an apparatus arranged so that a rapid stream of dry nitrogen sweeps the surface of the reactants. If the nitric acid is added very slowly, there is formed, in the bottom of the reactor, a reddish-black solid substance which upon the further, more rapid addition of nitric acid bursts into flame. From these observations and others to be discussed later, it appeared that a solid phase was formed prior to the ignition of the dicyclopentadiene-WFNA reaction. To establish whether or not a solid phase is formed, the reaction was studied photographically; a Western Electric Fastax high-speed motion-picture camera recorded the progress of the reaction. The photographs comprising Fig. 1 are from portions of the film taken at 1500 frames per sec; that is, approximately 0.0007 sec between frames.

The hypergolic reaction was produced by allowing fine streams of the reactants to impinge on a v-shaped metallic surface. To eliminate any effect of the presence of air on the reaction and to prevent the clouding of the metallic surface by the considerable quantity of vapor evolved during the reaction, the experiment depicted in Fig. 1 was carried out under a rapidly moving atmosphere of dry nitrogen which swept across the surface of the reactants. The sequence in Fig. 1 clearly shows the formation of the solid phase (the dark area to the right of the injection apparatus) and that ignition occurs on the surface of that solid material. The

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[1] Abstracted from a thesis by Charles H. Trent, submitted to the faculty of the Graduate School of Purdue University in partial fulfillment of the requirements for the degree of doctor of philosophy, June, 1951.

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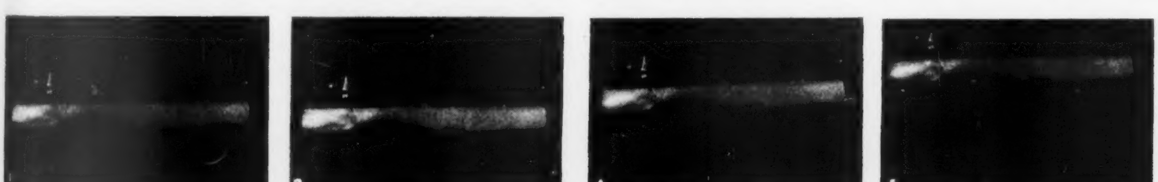
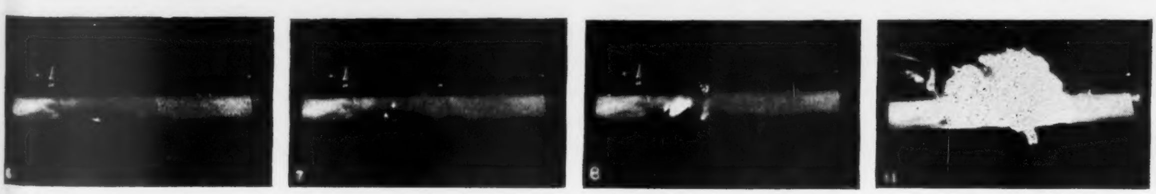


Frame before ignition | Start of ignition | 0.0007 sec later | 0.0021 sec after ignition



0.0035 sec after ignition | 0.0056 sec after ignition | 0.0098 sec after ignition | 0.025 sec after ignition

FIG. 1. PHOTOGRAPHS OF THE HYPERGOLIC REACTION BETWEEN DICYCLOPENTADIENE AND NITRIC ACID. GASES REMOVED BY MOVING ATMOSPHERE OF DRY NITROGEN. FILM SPEED 1500 FRAMES PER SEC.

point of ignition can be seen as a faint light spot in the center of the solid (Fig. 1, frame 3) which grows in intensity in succeeding frames. Approximately 0.0056 sec after ignition has occurred, the combustion of the residual vapor above the solid phase was initiated (Fig. 1, frame 7). A second point of ignition on the surface of the solid phase is seen to occur in Fig. 1, frame 8.

Another series of photographs taken of the reaction between dicyclopentadiene and nitric acid resemble those of Fig. 1, differing only that the ignition occurs at a different place on the surface of the solid phase. If the reaction is carried out in still air, the ignition still occurs on the surface of the solid but the flame is rapidly propagated throughout the vapor above the surface of the solid.

That the dark area on the metallic surface shown in Fig. 1 is actually a solid phase has been demonstrated by the actual isolation of solid compounds during controlled reactions between dicyclopentadiene and nitric acid.

Chemical Nature of the Reaction

The dropwise addition of a molar equivalent of nitric acid (99 per cent) to dicyclopentadiene immediately and exothermically forms an amorphous dark-red solid insoluble in the original hydrocarbon. The solid is sufficiently stable to permit isolation and chemical examination. Tests showed that the solid compound or mixture of compounds formed are not shock sensitive, but when rapidly heated the solids burst into flame. Moreover, the addition of about 10 drops of concentrated nitric acid to a small amount of the solids heated to 100 C will cause combustion to occur.

The reaction between dicyclopentadiene and nitric acid has also been conducted in dilute solutions employing carbon tetrachloride and petroleum ether as the diluents. From those reactions sufficient amounts of solid material were isolated for further study. It was found that the quantity of solids precipitated during the reaction increases as the molar ratio of nitric acid to dicyclopentadiene increases. The physical properties of the solid phase also appear to be altered, Table 1. In addition to solid compounds, very unstable liquid products were also isolated. The latter explode upon attempted distillation at atmospheric pressure.

TABLE 1 SOLID FORMATION DURING REACTION BETWEEN DICYCLOPENTADIENE AND NITRIC ACID

No.	Dcp, g	HNO_3, g	Solid formed, g	D.p., C
203	13.2	6.3	3.8 (I)	155–160
204	13.2	12.6	9.6 (VI)	172
206	13.2	18.9	10.4 (XI)	210

Insufficient research was conducted on the chemical properties of the solid compounds of Table 1 to establish their exact chemical structure. The chemical behavior of solids I and VI, however, leads to the conclusion that the solid compounds contain the nitro group and the nitrate ester group. The elemental analyses of I and VI are presented in Table 2.

TABLE 2 ELEMENTAL ANALYSES OF SOLIDS DERIVED FROM REACTION BETWEEN DICYCLOPENTADIENE AND NITRIC ACID

Compound	C, %	H, %	N, %	O, %
I	53.79	4.69	8.19	33.33
VI	51.80	4.73	9.05	34.48
VI	51.62	4.85	8.98	

Because of their thermal instability it was not possible to determine the molecular weights of Solids I and VI by the Rast method.

The ability of dicyclopentadiene to polymerize when treated with strong acids, the insolubility, and the amorphous character of Solids I and VI indicate that the latter substances are probably highly nitrated polymers. Dicyclopentadiene treated with concentrated sulfuric acid under identical experimental conditions as with nitric acid is readily and exothermically polymerized to a hard reddish-black substance. If several drops of sulfuric acid are added to dicyclopentadiene prior to the addition of nitric acid, the ensuing combustion is markedly more vigorous than in the absence of sulfuric acid. The effect of the presence of a strong polymerization catalyst on the rate of the hypergolic reaction can be inferred by comparing the values of ignition delay for various mixtures of sulfuric and nitric acids. Dicyclopentadiene when reacted with anhydrous nitric acid exhibits an ignition delay of 0.032 sec. The addition of 5 per cent by weight of concentrated (96.7 per cent) sulfuric acid to anhydrous nitric acid reduces the ignition delay to 0.026 sec; the addition of 15 per cent by weight of sulfuric acid further reduces the ignition delay to 0.011 sec. However, the addition of 5 per cent by weight of another polymerization catalyst, cumene hydroperoxide, served merely to dilute the dicyclopentadiene since, when reacted with anhydrous nitric acid, the ignition delay was increased to 0.048 sec.

Experimental

Preparation of Solids I and VI: Solid I was prepared by the dropwise addition of 0.1 mol of 99 per cent nitric acid to 0.1 mol of dicyclopentadiene dissolved in 60 ml of carbon tetrachloride and contained in a 250-ml beaker equipped with a mechanical stirrer. The addition of nitric acid was regulated to keep the temperature of the reaction between 15–20 C; if the acid is added too rapidly the reaction temperature rises rapidly and an explosive decomposition reaction may occur. The reddish-brown solid precipitated during the addition of nitric acid was removed by filtration, and then washed with fresh portions of carbon tetrachloride and anhydrous ethyl ether to remove any traces of occluded nitric acid. The dry weight of Solid I was 3.8 g; decomposition point 155–160 C.

The procedure employed for preparing Solid I was utilized for the preparation of Solid VI. The yield of solid VI was 9.6 g when 0.2 mol of nitric acid was reacted with 0.1 mol of dicyclopentadiene. The decomposition point of Solid VI is 172 C.

Solids I and VI do not possess melting points as ordinarily determined because when heated slowly they gradually decompose. However, there is a definite temperature at which Solids I and VI, when dropped onto a hot metallic surface, immediately decompose giving off a dense white vapor and oftentimes bursting into flame. Below the decomposition temperature, Solids I and VI slowly darken becoming black, but there is no other outward indication of their decomposition.

Dicyclopentadiene: The dicyclopentadiene used in all experiments with nitric acid was obtained as a commercial research sample. The material as received was a straw-colored liquid. Prior to use, the dicyclopentadiene was distilled under reduced pressure to yield a water-white liquid boiling at 64 C (19 mm); d_4^{26} 0.9395; n_D^{26} 1.496; molecular refraction calcd 40.9, found 41.1.

Acknowledgment

The authors wish to acknowledge Project SQUID, the Office of Naval Research, and the Office of Air Research whose financial support made this work possible.

Single Flow Jet Engines—A Generalized Treatment

(Continued from page 126)

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Letters to the Editor

This section of the Journal is open to letters not exceeding 600 words in length (or one and one-half columns) devoted to brief research reports or technical discussions of papers previously published. Such letters are published without editorial review, usually within two months of the date of receipt. The style and manner of submission of letters are the same as for regular contributions. (See inside back cover.)

Jet Propulsion News

By C. F. WARNER, Purdue University, *Associate Editor*

Missiles

THE Navy Department has placed an order with Consolidated Vultee to start producing its Terrier guided missile for shipboard launching against aircraft. It also ordered Douglas Aircraft to begin production of its air-to-air guided missile Sparrow.

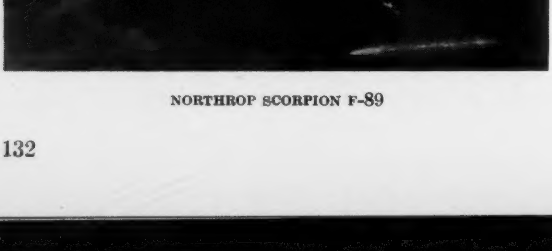
The Convair Terrier is a two-stage rocket developed by the Bureau of Ordnance. The solid-propellant missile weighs more than a ton and a half, including the booster and warhead and has a range of approximately ten miles. The Douglas Sparrow, weighing 280 lb including its warhead, was developed by the Bureau of Aeronautics and is designed to be used by carrier-based aircraft in action against enemy aircraft at ranges of approximately five miles.

Aircraft

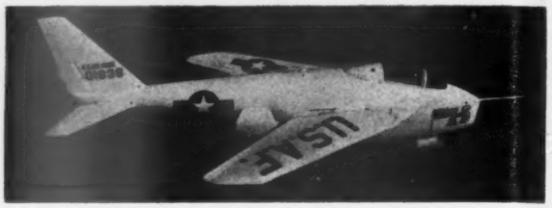
ON July 3rd, the Navy announced that the Douglas Skyrocket, a supersonic research plane, has flown faster and higher than any other known craft. The plane was released from its mother ship, a B-29, over the California desert.

Bell Aircraft Corporation has just delivered its X-5 jet-propelled airplane to the Edwards Air Force Base, Muroc, Calif., where it will be flight tested. The X-5 is the first airplane to be equipped with movable wings permitting the pilot to change the degree of sweepback during flight. For take-off, early climb, and landing, the wings are moved forward to obtain best performance, whereas the wings are moved rearward when great speed at high altitudes is desired.

Northrop Aircraft, Inc., is now in quantity production of its all-weather interceptor, the Scorpion F-89. The aircraft, carrying a crew of two, is powered by two Allison J-35 engines equipped with afterburners. The F-89 has a wing span of 50 ft, is approximately 50 ft long, and is designed for a gross weight of over 30,000 lb. The Air Force has announced that the Scorpion is



NORTHROP SCORPION F-89



BELL X-5 PIONEER IN STUDY OF MOVABLE WINGS

capable of operating at speeds in the 600-mile-per-hour range and of operating at altitudes in excess of 40,000 ft. The air-borne electronic equipment enables the Scorpion to fly in darkness, fog, and storms, and to locate hostile airplanes under adverse weather conditions.

Turbojet Engines

THE General Electric Company has announced the completion of its new turbojet engine, the J-47-GE-21, which although the same size as its J-47 (rated in excess of 5200-lb thrust), is far more powerful. The new "21" is an all-weather engine with anti-icing features and high-altitude starting characteristics. It may be equipped for either water/alcohol injection or afterburning for greatly added thrust power for short periods. A tail-pipe variable jet nozzle may be added to obtain maximum efficiency. The engine has a so-called "cannular" type of combustor consisting of a single combustion space containing individual cans. The General Electric Company considered development of the compressor so significant that it bestowed its highest employee honor, the Coffin Award, upon two of the engineers responsible for the compressor's design, Russell Hall and William Cornell of the Aircraft Gas Turbine Division.

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PRATT and Whitney's newest and largest jet engine, the J-57, was recently test flown in a B-50. The B-50's four Wasp Major engines were idled and the J-57 turbojet engine alone flew the B-50 at 370 mph. Frederick Brant Rentschler, president of United Aircraft Corporation, said that Pratt and Whitney's new J-57 has more power than any other turbojet engine ever flown and has gone a long way in reducing the enormous fuel consumption of jet engines.

New Facilities

A NEW plant for producing jet engine components will be built on the outskirts of Columbus, Ohio, by the

Westinghouse Electric Corporation. The plant will be the largest single plant yet built by the company.

James Forrestal Research Center

ANNOUNCEMENT was made Monday, May 28, 1951, of the formation of a committee to raise $2,000,000 in endowment funds for the James Forrestal Research Center at Princeton University, Princeton, N. J., a memorial to the late Secretary of Defense.

Initial members of the committee, to be composed of friends and close associates of Mr. Forrestal, were announced at a dinner at the Union Club given by Judge Robert P. Patterson, the committee chairman.

In addition to the $1,500,000 provided by Princeton for the purchase of the property, $2,000,000 is needed for endowment to permit tenure of appointments to key scientists and for the improvement of facilities.

The Center, devoted to co-operative enterprise in the fields of engineering, mathematics, chemistry, and physics, will bring together eminent scientists and skillful research staffs to solve problems of prime importance to our national defense, transportation, and industry. Defense agencies have testified to their vital interest in the project by pledging all possible assistance and initial support to the program, and projected research contracts assure capacity operation of the Center's facilities

The James Forrestal Research Center is housed in two three-story buildings and 16 auxiliary buildings located south of U. S. Route 1, about two and a half miles from the center of Princeton campus. The projects in progress, some of which are now being moved to the new location, are co-ordinated with the research and defense program of the nation. Among the projects planned for the new center are several important researches in jet and rocket propulsion, supersonic aerodynamics, combustion, and new fuels.

Personalities

THE appointment of Thomas G. Lanphier, Jr., as assistant to the president of Consolidated Vultee Aircraft Corporation, has been announced by President La Mott T. Cohu. Mr. Lanphier will participate in policy planning in the guided-missile field of the company. Convair will build a new plant in Pomona, Calif., for the manufacture of guided missiles.

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The appointment of John H. Wallace, Jr., as assistant director of the James Forrestal Research Center at Princeton University, was announced by President Harold W. Dodds. He will assist with the development and administration of the laboratory facilities and with the organization of the joint services. The director of the Center is Prof. Daniel C. Sayre, formerly chairman of the Department of Aeronautical Engineering.

The board of directors of Reaction Motors, Inc., announced that Mr. Lovell Lawrence, Jr., formerly president of the corporation, has been appointed chairman of the board of directors. Mr. Raymond W. Young, former vice-president of engineering for the Wright Aeronautical Corporation, and later for Reaction Motors, Inc., was appointed president and general manager.

American Rocket Society News

By A. F. BOCHENEK, *Associate Editor*

First ARS Meeting in Canada Attracts Good Attendance

THE technical sessions and the exhibit of ARS literature sponsored by the American Rocket Society at the semi-annual meeting of The American Society of Mechanical Engineers held at Royal York Hotel, Toronto, Ontario, Canada, June 11–14, 1951, won many Canadian friends for the Society, according to C. W. Chillson, Vice-President ARS, and chairman of the program committee.

Both sessions were well attended and in addition to prepared discussion, extemporaneous comment on various aspects of the papers added much to the interest of the meetings.

Exhibit Attracts Attention

The exhibit of ARS literature gave an opportunity for many Canadians to learn of the work of the Society, as they inspected issues of the JOURNAL and asked questions about the Society's organization, history, and leaders.

Interest in the Society was also evident from newspaper coverage of ARS sessions, and from radio news comment carried on two broadcasts.

At the morning session, attended by 35, Lowell N. Randall, rocket test supervisor, Curtiss-Wright Corporation, Caldwell, N. J., discussed the operation and details of construction of the cavitating Venturi for rocket applications. Mr. Randall said that the cavitating Venturi has been ap-

plied extensively as a simple and accurate flow-control device, and also has been used for temperature measurement purposes.

The second paper was "Equipment for Handling High-Strength Hydrogen Peroxide," by Noah S. Davis, Jr., manager, special products department, and John H. Keefe, Jr., project supervisor, Buffalo Electro-Chemical Company, Inc., Buffalo, N. Y.

Hydrogen Peroxide Equipment

The authors described the search made for the most satisfactory equipment for handling hydrogen peroxide in storage and propellant systems. Compatibility tests were run, they reported, on plastics, and various metals. They also disclosed useful information about equipment design and preparation of equipment for handling this rocket fuel.

The final paper of the morning session was "Optical Methods of Rocket-Motor Evaluation," by Kurt R. Stehling, rocket-research engineer, Bell Aircraft Corporation, Buffalo, N. Y.

Mr. Stehling described various methods tried by his company for measuring and analyzing thrust-chamber phenomena, particularly the infrared-pyrometry and line-reversal methods of temperature measurement. He also commented on spectrometric and spectrophotometric methods as compared with conventional analysis methods.

Uncooled Rocket Motors

At the afternoon session attended by 75, three papers were presented under the general subject of uncooled rocket motors. The Society was fortunate in having as chairman for this session, Lt. Col. R. A. Jones, AMC Power Plant Laboratory, Wright-Patterson Air Force Base, Dayton, Ohio.

William R. Sheridan, rocket section engineer, Bell Aircraft Corporation, Buffalo, N. Y., opened the session with a paper on "Materials for Use in Uncooled Liquid-Propellant Rocket Motors." He described the desirable properties for materials used as the refractory liners, the load-carrying metal outer shell, and the intermediate layer of insulating cement in construction of uncooled liquid-propellant rocket combustion chambers.

Ceramic Linings

Ceramic linings were discussed in further detail in the second paper, "The Success of Ceramic-Lined Rocket Motors," by H. Zane Schofield, supervisor, and W. H. Duckworth, assistant supervisor, ceramic division, Battelle Memorial Institute, Columbus, Ohio. The authors reported tests which showed that sound practices in engineering could remove the observed shortcomings of such linings in most cases. It was their opinion that ceramics had a definite future in rocketry if their properties were properly exploited.

In the final paper of the afternoon, "Silicon Carbide Linings for Uncooled Rocket Motors," by Kenneth C. Nicholson, supervising engineer, research and development department, The Carborundum Company, Niagara Falls, N. Y., silicon carbide as a lining material was discussed. Mr. Nicholson outlined the properties of this material and commented on available sizes and shapes and methods of fabrication and installation.

Second International Congress on Astronautics Meets in London, Sept. 3–8

THE American Rocket Society will be represented at the Second International Congress on Astronautics by Andrew G. Haley, general counsel ARS, and Lieut. Comdr. F. C. Durant III, members of the ARS Board of Directors.

The congress, which will take place in Caxton Hall, London, England, Sept. 3–8, 1951, will complete plans for an International Astronautical Federation and present a program of papers on the general theme of the Earth-Satellite Vehicle by international astronautical authorities.

ARS Contribution to Program

The American Rocket Society is sponsoring one technical paper. It is: "The Importance of Satellite Vehicles in Interplanetary Flight," by Dr. W. von Braun.

The first part of the Congress will be devoted to procedural matters pertinent to organizing the International Astronautical Federation. These sessions will be open to official delegates only. Draft proposals for the new body have already been considered by the ARS Board of Directors and Mr. Haley, as official spokesman for the Society, will convey the ARS point of view.

It is planned to inaugurate the new Federation on Tuesday, September 4. The next day will be devoted to detailed discussion for plans for the 1952 Congress and a program for the Federation, which will probably include such matters as standardization projects for symbols and nomenclature.

16 Papers Scheduled

The technical sessions will begin on Thursday morning. Sixteen papers are scheduled for presentation in four sessions. While the public will be invited, the program is expected to have little interest for laymen.

The program will conclude with a popular technical symposium on Saturday afternoon at which spokesmen from the various national delegations will give short talks explaining their ideas on interplanetary flight. A public statement will also be made giving the results of the preceding Congress sessions.

Some of the other papers on the technical program are: Applications of Satellite Vehicles, by E. Birch Andersen (Denmark); The Ascent of Satellite Vehicles, by D. F. Lawden (Britain); Views on the Ascent of the Earth Satellite Vehicle, by Ing. Janson (Germany); Start, Return, and Landing of an Optimum Satellite Step Rocket, by Dipl. Ing. H. Kuhme (Germany).

As the final event of the Congress, delegates will be entertained at dinner at the St. Ermin's Hotel, tendered by the British Interplanetary Society.

Student Papers Heard at New York Section Meeting

UNDERGRADUATE research projects carried out by students at Pratt Institute, Brooklyn, N. Y., and at the College of the City of New York, New York, N. Y., were described at a meeting of the New York Section of the American Rocket Society held at the Engineering Societies Building, New York, N. Y., May 25, 1951. The meeting was arranged by Major James R. Randolph, a member of the Board of Directors of the New York Chapter. Fifty members and guests were present. Major Randolph presided.

Design and construction of an improved nozzle and test section and the use of superheaters to obtain a high Mach number flow in a supersonic wind tunnel were described by Roy O. Bohner and Vincent P. Buschimi, Pratt Institute students.

The second talk was by V. A. Chobotov and Howard Dobias of Pratt Institute. They described the design of a pulse jet to be used for static testing of fuels, nozzles, and other factors involved in pulse jets.

Ralph A. Benson of Pratt Institute described his work on a static test rocket for gaseous fuels. He pointed out that his rocket could be used for testing propellants and mixing devices. It was fired upward, was water cooled, and was designed to be used with an ordinary household scale to measure thrust.

The last project described was by Howard Macklis, Sheldon Newberger, and Harris L. Shapiro of the College of the City of New York. They demonstrated how the theory of gas turbines is applied to a specific design problem. Their design gave a thrust of 2600 lb. They showed also how dimensions and angles of blades and other details not derived from thermodynamics were calculated.

Indiana Section Completes Memorable First Year

THE Indiana Section of the American Rocket Society climaxed a memorable first year of operation by an "Acceleration Banquet" held in the North Ballroom of the Purdue Memorial Building, West Lafayette, Ind., May 24, 1951. A distinguished audience of nearly 100 persons which included officials of the University, Lt. Gov. John A. Watkins of Indiana, and H. R. J. Grosch, president of the American Rocket Society, heard Under-Secretary of the Navy Dan A. Kimball talk on the history and present status of rocket development. (Mr. Kimball has since been named Secretary of the Navy.) J. P. Layton, president of the Indiana Section, presided.

Mr. Layton introduced charter members of the Section, newly elected officers, and members of the Board of Directors of the Section and reviewed the program of the past season.

Purdue Officials Speak

Following a brief introduction by President Layton, R. B. Stewart, vice-president of Purdue University, who represented Purdue's president, F. L. Hovde, expressed the good wishes of the University for the continued success of the Section and congratulated the Section for its recognition by the faculty as a university activity. He was followed by the Honorable John A. Watkins, lieutenant-governor of Indiana, who represented Governor Schricker. Lt. Gov. Watkins spoke highly of the scientific work of the ARS. He was followed by A. A. Potter, dean of the University's School of Engineering, who reviewed the development of engineering education in the United States, particularly the contributions of the land-grant colleges. Dean Potter cited many "firsts" in engineering education at Purdue, and then stressed the importance of the humanities as well as technology for the development of good character and responsible citizenship.

ARS President Present

President Grosch spoke briefly to review the development of the American Rocket Society from its early days of backyard experimentation to its present status as one of the leading technical organizations working in the field of rocket and jet-propulsion technology. Rocketry, he commented, was a young man's field and one in which talented hard-working young men can progress far in a few years. President Grosch announced expansion of the ARS JOURNAL both in format and in number of issues. He spoke appreciatively of the Society's joint activities with The American Society of Mechanical Engineers and the Institute of the Aeronautical Sciences.

SECRETARY OF THE NAVY DAN A. KIMBALL, MAIN SPEAKER AT THE ARS INDIANA SECTION DINNER HELD MAY 24, 1951

The Society's Board of Directors, he said, was working to increase aid on the national level to the ARS Sections and to organize Sections in centers of rocket research and industry not now being served by Sections. The ARS Fall meeting at Minneapolis, Minn., he added, would give members in the Midwest an opportunity to attend a major meeting of the Society. He invited all to attend and to participate in the discussion of the program of interesting papers.

Promising Progress

Following a welcome from Capt. J. C. Woelfel of the Purdue Naval Reserve Officers Training Corps, President Layton introduced the Honorable Dan A. Kimball, Undersecretary of the Navy. In spite of promising progress in the field of rockets and jet propulsion, Mr. Kimball said, a great deal remains to be done. Ever-increasing technological challenges face members of the ARS. As development progresses, rockets will become more and more useful instruments of national defense and civil progress, he predicted.

New Officers Elected

At a meeting of the Indiana Section on May 3, 1951, the following officers were elected: *President*, David McNay; *Vice-President*, Clair M. Beighley; *Secretary*, Vincent Capasso; *Treasurer*, Ralph Kester; *Board of Directors*, M. J. Zucrow, W. Gilliland, Arthur Wiggins, Gladys Geiger, and J. P. Layton.

ARS Southern California Section Holds Closed Meeting

THE following officers have been elected for the 1951–1952 season by the Southern California Section of the American Rocket Society: *President*, R. B. Canright, Jet Propulsion Laboratory, California Institute of Technology; *Vice-President*, B. L. Dorman, Aerojet Engineering Corporation; *Secretary-Treasurer*, R. J. Lodge, North American Aviation, Inc. Directors elected were as follows: C. C. Ross, Aerojet Engineering Corp.; L. G. Dunn, Jet Propulsion Laboratory, California Institute of Technology; J. F. Manildi, University of California, Los Angeles, Calif.; G. D. Brewer, Hughes Aircraft Corp.; W. T. Cox, USN, Bureau of Aeronautics; W. E. Zisch, Aerojet Engineering Corp.; E. G. Crofut, Aerolab Development Company.

The Southern California Section held a highly successful closed dinner meeting at the Pasadena Athletic Club on April 24, 1951. The title of the meeting was "Symposium on Propellant Selection." President Canright led the discussion by a panel of rocket authorities which included: Tom Dixon, North American Aviation, Inc.; Don L. Armstrong, Aerojet Engineering Corp.; Leland G. Cole, Jet Propulsion Laboratory, California Institute of Technology; David A. Young, Aerojet Engineering Corp.; and T. E. Myers, North American Aviation, Inc.

The meeting was of such interest that few of the nearly 200 in the audience left before 11:30 p.m. The meeting was also distinctive because it was the largest classified meeting ever held by the Section. In addition to local members and guests, all local military services were well represented. The meeting closed with the showing of a film from North American Aviation, Inc., on testing propellants in a small engine.

Interesting Papers Planned for 1951 ARS Fall Meeting

THE American Rocket Society will hold its 1951 Fall Meeting in conjunction with The American Society of Mechanical Engineers at the Radisson Hotel, Minneapolis, Minn., Sept. 25–28, 1951.

The ARS part of the program will be a session on Thursday afternoon, September 25, during which two papers will be presented. The first will be by Clark E. Thorp, chairman of chemistry and chemi-

cal engineering research, Armour Research Foundation, Illinois Institute of Technology, Chicago, Ill. Dr. Thorp's subject will be "The Properties and Production of Concentrated Gaseous and Liquid Ozone."

The second paper by Stanley V. Gunn, Purdue University Rocket Laboratory, Lafayette, Ind., will be on "The Effects of Several Variables Upon the Ignition of Hypergolic Fuels Oxidized by Nitric Acid." Mr. Gunn has divided his paper into three parts covering (1) description of electronic timer and reaction apparatus employed for open-cup tests; (2) ignition-lag data obtained from open-cup tests; and (3) a description of the reaction apparatus to be employed for ignition-lag determinations within a small rocket motor. He will also report data which show the effects of propellant temperature and water content of the oxidizer on ignition lag of several hypergolic fuels, when oxidized with nitric acid.

Members of the Society, especially those residing in the Midwest, are urged to participate in the Fall Meeting. Engineers who are not ARS members but who are working in the rocket industry are also invited to the ARS session.

New York Section Plans Fall Meetings

THE ARS New York Section will open its 1951-1952 series of meetings at the Engineering Societies Building, New York, N. Y., Sept. 21, 1951, when Edward F. Chandler, engineer and inventor, of Brooklyn, N. Y., will talk on "Adapting the Rocket to Modern Warfare."

Mr. Chandler will review some phases of the work he is doing on a method of firing rocket-type ammunition from breech-loading firearms. He is of the opinion that the tactical uses of the rocket can be increased considerably by designing a rocket as a self-contained round or "fixed ammunition."

This would make available light, highly mobile arms capable of being rapidly served and fired either manually or automatically.

The regular meeting will be held on Friday, Oct. 19, 1951, at 8 p.m. The speaker will be Dr. R. W. Porter, Division Engineer, Special Project Division, General Electric Company, Schenectady, N. Y. The subject of Dr. Porter's talk will be "Failures in the V-2 Program." Dr. Porter will discuss the V-2 firing with particular regard to material difficulties which prevented accomplishment of specific objective desired.

ARS Junior Award Competition

THE distinction of receiving an American Rocket Society Junior Award carries with it a recognition of meritorious achievement early in one's professional career. Papers are now being accepted at the New York office for consideration for the 1951 award. The award and medal will be presented at the Annual Convention to be held in November. To be considered, papers must be received not later than Oct. 20, 1951.

The winning paper will be judged mainly on the basis of content which should reflect original thought and effort. The age of authors of papers must not be greater than 25 years. Papers should be on standard size paper, typewritten, and clearly marked, "Submitted for Junior Award Competition." Send papers to: Secretary, American Rocket Society, 29 West 39th Street, New York 18, N. Y.

Technical Literature Digest

By H. S. SEIFERT, California Institute of Technology, *Associate Editor*

CONTRIBUTORS: F. C. Gunther, D. Altman, S. S. Penner, R. B. Canright, D. I. Baker, H. S. Siefert

EDITOR'S NOTE:

The following collection of references is not intended to be comprehensive, but is rather a selection of the most significant and stimulating papers which have come to the attention of the contributors. The reader will understand that a considerable body of literature is unavailable because of security restrictions. We invite contributions to this department of references which have not come to our attention, as well as comment on how the department may better serve its function of providing leads to the jet-propulsion applications of many diverse fields of knowledge.

Inasmuch as this represents the first issue of the Literature Digest department, some references dated even more than one year ago have been listed. In future issues, the contributors plan to devote the space primarily to current listings.

Book Review

SERVOMECHANISMS AND REGULATING SYSTEM DESIGN, by Harold Chestnut and Robert W. Mayer, John Wiley & Sons, Inc., New York, N. Y., vol. I, xiii + 505 pp. $7.75.

THIS book is a carefully executed engineering text on a subject of vital concern to those interested in the guidance and control of missiles. Three of its fourteen chapters review useful mathematical background, including applications of complex numbers and the Laplace transform. There is a brief review of certain aspects of network theory, followed by a thirty-page discussion of stability criteria for control systems. Chapters 7 through 12 discuss many examples of single closed-loop control systems (including on p. 190 the control of a missile in vertical flight) and the use of graphical techniques for their analysis and synthesis. Multiple-loop control systems are discussed in chapter 13, and the correlation between transient and steady-state performance of servomechanisms is examined in the last chapter.

There is a fifty-page set of problems at the end of the book, a very valuable feature. This book is based on extensive teaching and engineering experience of the authors and their colleagues at the General Electric Co., and is a thorough, sound, and quantitative approach to an important subject. Volume II of the set will extend the subject to more detailed consideration of practical design problems, synthesis of systems, "noise," nonlinearities, and the like.

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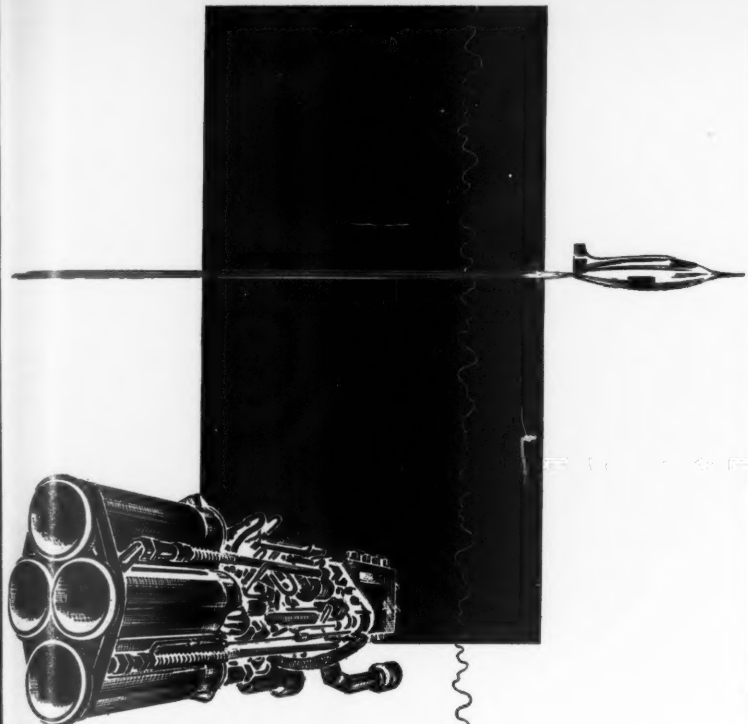

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The Society carries out its aims by the following methods:

Represents the jet propulsion profession before the public, promotes interest in jet propulsion and its applications, encourages the formation of regional chapters and student branches where technical conferences may be held.

Publishes and disseminates news, experimental results, and technical information to its members, through the medium of its Journal.

Encourages and aids those actively working in this field by organizing national and regional technical meetings where contributed and invited papers are presented and discussed.

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